CHAPTER 3

THE COUPLED MODELING OF NINE-PHASE IPM MACHINES

3.1 Introduction

In this chapter full order models are generated for nine-phase IPM machines for different stator connections. The coupled models use the basic geometry of the machine, windings functions, airgap function and permanent magnet flux linkage [83]. In this chapter the equations of the coupled models are derived in the rotor reference frame. Then using the clock diagram and the airgap function of the machines, the turn and winding function of the stator windings are generated. Using the winding and turn functions, the machine inductances are generated as functions of the rotor angle. The stator inductances are then transformed to the rotor reference frame. The flux linkages of the permanent magnets, which link the stator phases of the machines, are also modeled and transformed to the rotor reference frame. Finally using the inductances and flux linkages, the machines are modeled in rotor reference frame and the models are verified with simulation results. The major contribution of this chapter is to generate full order models that are capable to predict both high frequency and low frequency behaviors of the machine. The first section of this chapter is the modelling of a nine phase IPM machine which has one single star point. The machine inductances of the single star machine in q and d axis of the rotor reference frame are also calculated using fainant element method magnetics (FEMM). After this the full order model of the same machine with three isolated stars (symmetrical) is derived and finally the full order model of an asymmetrical triple star machine is derived. Using the models of this chapter, both the high frequency and low frequency simulation of the machines can be done using one single model. The models are simulated using MATLAB/Simulink and the simulation results are presented for each machine.

3.2 Coupled Modeling of the Single-Star Nine-Phase IPM

This model which is a full order model based on the machine geometry is an accurate model that can be used to study most details of the machine behavior. In this model there would be limited simplifying assumption in terms of the winding functions and the other parts of the machine structure. Phase voltages for phases 'a', 'b', 'c', 'd', 'e', 'f', 'g', 'h' and 'i' in 9-phase IPM machine (presented in Figure 3.1 (a)) are given as equation (3.1). In these equations 'I_x' is the current of phase 'x' and '$p\lambda_x$' is the derivation of the flux linkage seen from the phase 'x' and the term 'r_s' also represents the stator resistance for each phase of the stator [83].

$$
\begin{aligned}
V_a &= r_s I_a + p\lambda_a \\
V_b &= r_s I_b + p\lambda_b \\
V_c &= r_s I_c + p\lambda_c \\
V_d &= r_s I_d + p\lambda_d \\
V_e &= r_s I_e + p\lambda_e \\
V_f &= r_s I_f + p\lambda_f \\
V_g &= r_s I_g + p\lambda_g \\
V_h &= r_s I_h + p\lambda_h \\
V_i &= r_s I_i + p\lambda_i
\end{aligned}
\qquad (3.1)
$$

The machine is a 4-pole nine-phase with concentrated windings. The machine has 36 slots. The slot angular pitch can be calculated as:

$$\gamma = \frac{180 \times P}{36} = \frac{180 \times 4}{36} = 20 \ \ (Degree) \tag{3.2}$$

The slot between phases and the full coil pitch can be calculated as:

$$Slot\ Between\ \ Phases = \frac{Phase\ \ Shift\ \ Between\ \ \ Phases}{\gamma} = \frac{40}{20} = 2 \tag{3.3}$$

$$Full\ Coil\ Pitch = \frac{Number\ of\ Slots}{P} = \frac{36}{4} = 9 \tag{3.4}$$

Since the machine has concentrated windings then the belt is equal to 1. Using the equations (3.2) to (3.4) the winding scheme of the machine can be generated as Table 3.1.

Table 3.1 The winding scheme of nine-phase machine.

A^+	A^-	B^+	B^-	C^+	C^-
1	10	3	12	5	14

A_-	A_+	B_-	B_+	C_-	C_+
10	19	12	21	14	23

A^+	A^-	B^+	B^-	C^+	C^-
19	28	21	30	23	32

A_-	A_+	B_-	B_+	C_-	C_+
28	1	30	3	32	5

D^+	D^-	E^+	E^-	F^+	F^-
7	16	9	18	11	20

Table 3.1 Continued.

D_-	D_+	E_-	E_+	F_-	F_+
16	25	18	27	20	29

D^+	D^-	E^+	E^-	F^+	F^-
25	34	27	36	29	2

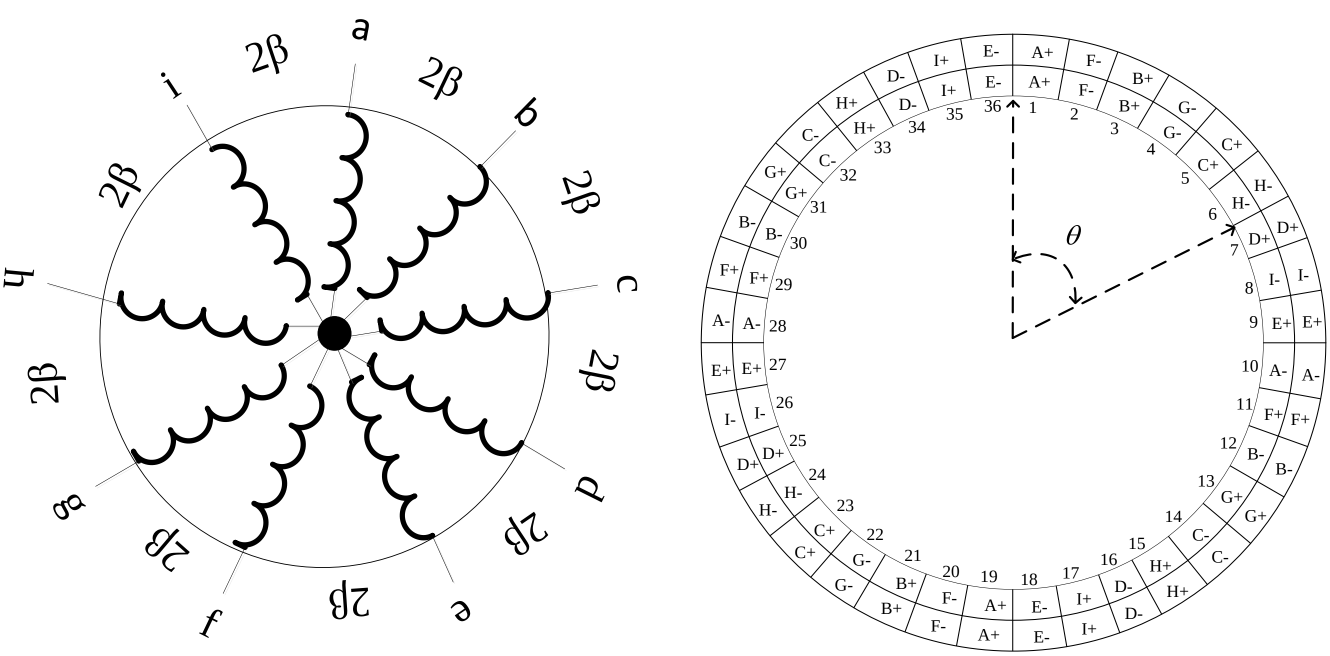
D_-	D_+	E_-	E_+	F_-	F_+
34	7	36	9	2	11

G^+	G^-	H^+	H^-	I^+	I^-
13	22	15	24	17	26

G_-	G_+	H_-	H_+	I_-	I_+
22	31	24	33	26	35

G^+	G^-	H^+	H^-	I^+	I^-
31	4	33	6	35	8

G_-	G_+	H_-	H_+	I_-	I_+
4	13	6	15	8	17



(a)

(b)

Figure 3.1: (a) The nine-phase machine, (b) The clock diagram of the nine-phase machine.

Using the Tables 3.1, the clock diagram can be drawn as Figure 3.1(b). The flux linkages of the phases can be represented as equation (3.5). In this equation the terms 'L_{xx}' represents the self-inductance of the phase 'x' and 'L_{xy}' represents the mutual inductance between phase 'x' and 'y'. Also the term 'λ_{pmx}' represents the flux linkage due to the permanent magnets of the rotor seen from the phase 'x' [83].

$$
\begin{aligned}
\lambda_a &= L_{aa}I_a + L_{ab}I_b + L_{ac}I_c + L_{ad}I_d + L_{ae}I_e + L_{af}I_f + L_{ag}I_g + L_{ah}I_h + L_{ai}I_i + \lambda_{pma} \\
\lambda_b &= L_{ba}I_a + L_{bb}I_b + L_{bc}I_c + L_{bd}I_d + L_{be}I_e + L_{bf}I_f + L_{bg}I_g + L_{bh}I_h + L_{bi}I_i + \lambda_{pmb} \\
\lambda_c &= L_{ca}I_a + L_{cb}I_b + L_{cc}I_c + L_{cd}I_d + L_{ce}I_e + L_{cf}I_f + L_{cg}I_g + L_{ch}I_h + L_{ci}I_i + \lambda_{pmc} \\
\lambda_d &= L_{da}I_a + L_{db}I_b + L_{dc}I_c + L_{dd}I_d + L_{de}I_e + L_{df}I_f + L_{dg}I_g + L_{dh}I_h + L_{di}I_i + \lambda_{pmd} \\
\lambda_e &= L_{ea}I_a + L_{eb}I_b + L_{ec}I_c + L_{ed}I_d + L_{ee}I_e + L_{ef}I_f + L_{eg}I_g + L_{eh}I_h + L_{ei}I_i + \lambda_{pme} \\
\lambda_f &= L_{fa}I_a + L_{fb}I_b + L_{fc}I_c + L_{fd}I_d + L_{fe}I_e + L_{ff}I_f + L_{fg}I_g + L_{fh}I_h + L_{fi}I_i + \lambda_{pmf} \\
\lambda_g &= L_{ga}I_a + L_{gb}I_b + L_{gc}I_c + L_{gd}I_d + L_{ge}I_e + L_{gf}I_f + L_{gg}I_g + L_{gh}I_h + L_{gi}I_i + \lambda_{pmg} \\
\lambda_h &= L_{ha}I_a + L_{hb}I_b + L_{hc}I_c + L_{hd}I_d + L_{he}I_e + L_{hf}I_f + L_{hg}I_g + L_{hh}I_h + L_{hi}I_i + \lambda_{pmh} \\
\lambda_i &= L_{ia}I_a + L_{ib}I_b + L_{ic}I_c + L_{id}I_d + L_{ie}I_e + L_{if}I_f + L_{ig}I_g + L_{ih}I_h + L_{ii}I_i + \lambda_{pmi}
\end{aligned} \tag{3.5}
$$

The voltages of equation (3.1) can be represented as:

$$V_{abcdefghi} = r_s I_{abcdefghi} + p\lambda_{abcdefghi} \tag{3.6}$$

The total Flux Linkage of the stator can be expressed as:

$$\lambda_{abcdefghi} = \lambda_{abcdefghi\_s} + \lambda_{abcdefghi\_pm} = L_{ss} I_{abcdefghi} + \lambda_{abcdefghi\_pm} \tag{3.7}$$

Where:

$$\lambda_{abcdefghi} = \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \\ \lambda_e \\ \lambda_f \\ \lambda_g \\ \lambda_h \\ \lambda_i \end{bmatrix}, \quad \lambda_{abcdefghi\_pm} = \begin{bmatrix} \lambda_{a\_pm} \\ \lambda_{b\_pm} \\ \lambda_{c\_pm} \\ \lambda_{d\_pm} \\ \lambda_{e\_pm} \\ \lambda_{f\_pm} \\ \lambda_{g\_pm} \\ \lambda_{h\_pm} \\ \lambda_{i\_pm} \end{bmatrix} \tag{3.8}$$

$$L_{ss} = \begin{bmatrix} L_{ls}+L_{aa} & L_{ab} & L_{ac} & L_{ad} & L_{ae} & L_{af} & L_{ag} & L_{ah} & L_{ai} \\ L_{ba} & L_{ls}+L_{bb} & L_{bc} & L_{bd} & L_{be} & L_{bf} & L_{bg} & L_{bh} & L_{bi} \\ L_{ca} & L_{cb} & L_{ls}+L_{cc} & L_{cd} & L_{ce} & L_{cf} & L_{cg} & L_{ch} & L_{ci} \\ L_{da} & L_{db} & L_{dc} & L_{ls}+L_{dd} & L_{de} & L_{df} & L_{dg} & L_{dh} & L_{di} \\ L_{ea} & L_{eb} & L_{ec} & L_{ed} & L_{ls}+L_{ee} & L_{ef} & L_{eg} & L_{eh} & L_{ei} \\ L_{fa} & L_{fb} & L_{fc} & L_{fd} & L_{fe} & L_{ls}+L_{ff} & L_{fg} & L_{fh} & L_{fi} \\ L_{ga} & L_{gb} & L_{gc} & L_{gd} & L_{ge} & L_{gf} & L_{ls}+L_{gg} & L_{gh} & L_{gi} \\ L_{ha} & L_{hb} & L_{hc} & L_{hd} & L_{he} & L_{hf} & L_{hg} & L_{ls}+L_{hh} & L_{hi} \\ L_{ia} & L_{ib} & L_{ic} & L_{id} & L_{ie} & L_{if} & L_{ig} & L_{ih} & L_{ls}+L_{ii} \end{bmatrix} \tag{3.9}$$

Using the transformation matrix of in equation (3.10) the voltages could be transformed to the rotor reference frame to obtain the various sequences of the voltages. This matrix is an expansion of the matrix presented in [154] to the nine-phase. In this equation *'C'* represents *'cos'* and *'S'* represents *'sin'*. The term 'α' is the phase shift between the stator phases of the machine and 'θ' is the reference frame angle.

$$T(\theta) =$$

$$\frac{2}{9}\begin{bmatrix} c(\theta) & c(\theta-\alpha) & c(\theta-2\alpha) & c(\theta-3\alpha) & c(\theta-4\alpha) & c(\theta-5\alpha) & c(\theta-6\alpha) & c(\theta-7\alpha) & c(\theta-8\alpha) \\ s(\theta) & s(\theta-\alpha) & s(\theta-2\alpha) & s(\theta-3\alpha) & s(\theta-4\alpha) & s(\theta-5\alpha) & s(\theta-6\alpha) & s(\theta-7\alpha) & s(\theta-8\alpha) \\ c(3\theta) & c3(\theta-\alpha) & c3(\theta-2\alpha) & c3(\theta-3\alpha) & c3(\theta-4\alpha) & c3(\theta-5\alpha) & c3(\theta-6\alpha) & c3(\theta-7\alpha) & c3(\theta-8\alpha) \\ s(3\theta) & s3(\theta-\alpha) & s3(\theta-2\alpha) & s3(\theta-3\alpha) & s3(\theta-4\alpha) & s3(\theta-5\alpha) & s3(\theta-6\alpha) & s3(\theta-7\alpha) & s3(\theta-8\alpha) \\ c(5\theta) & c5(\theta-\alpha) & c5(\theta-2\alpha) & c5(\theta-3\alpha) & c5(\theta-4\alpha) & c5(\theta-5\alpha) & c5(\theta-6\alpha) & c5(\theta-7\alpha) & c5(\theta-8\alpha) \\ s(5\theta) & s5(\theta-\alpha) & s5(\theta-2\alpha) & s5(\theta-3\alpha) & s5(\theta-4\alpha) & s5(\theta-5\alpha) & s5(\theta-6\alpha) & s5(\theta-7\alpha) & s5(\theta-8\alpha) \\ c(7\theta) & c7(\theta-\alpha) & c7(\theta-2\alpha) & c7(\theta-3\alpha) & c7(\theta-4\alpha) & c7(\theta-5\alpha) & c7(\theta-6\alpha) & c7(\theta-7\alpha) & c7(\theta-8\alpha) \\ s(7\theta) & s7(\theta-\alpha) & s7(\theta-2\alpha) & s7(\theta-3\alpha) & s7(\theta-4\alpha) & s7(\theta-5\alpha) & s7(\theta-6\alpha) & s7(\theta-7\alpha) & s7(\theta-8\alpha) \\ 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix} \quad (3.10)$$

$$,\alpha = \frac{2\pi}{9}, \theta = \textit{reference frame angle}$$

The different parts of the equation (3.6) can be transformed to the rotor reference frame as:

$$I_{qdor} = T(\theta_r) I_{abcdefghi} \Rightarrow I_{abcdefghi} = T(\theta_r)^{-1} I_{qdor} \tag{3.11}$$

$$V_{qdor} = T(\theta_r) V_{abcdefghi} \Rightarrow V_{abcdefghi} = T(\theta_r)^{-1} V_{qdor} \tag{3.12}$$

$$\lambda_{qdor} = T(\theta_r) \lambda_{abcdefghi} \Rightarrow \lambda_{abcdefghi} = T(\theta_r)^{-1} \lambda_{qdor} \tag{3.13}$$

Applying the variables of equations (3.11) to (3.13) in to the equation (3.6) results in:

$$V_{qdor} = T(\theta_r) r_{xs} T(\theta_r)^{-1} i_{qdr} + T(\theta_r) p T(\theta_r)^{-1} \lambda_{qdor} \tag{3.14}$$

The different terms of the equation (3.14) could be derived as:

$$T(\theta_r) p T(\theta_r)^{-1} \lambda_{qdor} = T(\theta_r) p T(\theta_r)^{-1} \lambda_{qdor} + T(\theta_r) T(\theta_r)^{-1} p \lambda_{qdor} \tag{3.15}$$

$$V_{qdor} = T(\theta_r) V_{abcdefghi} =$$

$$\frac{2}{9}\begin{bmatrix} c(\theta_r) & c(\theta_r-\alpha) & c(\theta_r-2\alpha) & c(\theta_r-3\alpha) & c(\theta_r-4\alpha) & c(\theta_r-5\alpha) & c(\theta_r-6\alpha) & c(\theta_r-7\alpha) & c(\theta_r-8\alpha) \\ s(\theta_r) & s(\theta_r-\alpha) & s(\theta_r-2\alpha) & s(\theta_r-3\alpha) & s(\theta_r-4\alpha) & s(\theta_r-5\alpha) & s(\theta_r-6\alpha) & s(\theta_r-7\alpha) & s(\theta_r-8\alpha) \\ c(3\theta_r) & c3(\theta_r-\alpha) & c3(\theta_r-2\alpha) & c3(\theta_r-3\alpha) & c3(\theta_r-4\alpha) & c3(\theta_r-5\alpha) & c3(\theta_r-6\alpha) & c3(\theta_r-7\alpha) & c3(\theta_r-8\alpha) \\ s(3\theta_r) & s3(\theta_r-\alpha) & s3(\theta_r-2\alpha) & s3(\theta_r-3\alpha) & s3(\theta_r-4\alpha) & s3(\theta_r-5\alpha) & s3(\theta_r-6\alpha) & s3(\theta_r-7\alpha) & s3(\theta_r-8\alpha) \\ c(5\theta_r) & c5(\theta_r-\alpha) & c5(\theta_r-2\alpha) & c5(\theta_r-3\alpha) & c5(\theta_r-4\alpha) & c5(\theta_r-5\alpha) & c5(\theta_r-6\alpha) & c5(\theta_r-7\alpha) & c5(\theta_r-8\alpha) \\ s(5\theta_r) & s5(\theta_r-\alpha) & s5(\theta_r-2\alpha) & s5(\theta_r-3\alpha) & s5(\theta_r-4\alpha) & s5(\theta_r-5\alpha) & s5(\theta_r-6\alpha) & s5(\theta_r-7\alpha) & s5(\theta_r-8\alpha) \\ c(7\theta_r) & c7(\theta_r-\alpha) & c7(\theta_r-2\alpha) & c7(\theta_r-3\alpha) & c7(\theta_r-4\alpha) & c7(\theta_r-5\alpha) & c7(\theta_r-6\alpha) & c7(\theta_r-7\alpha) & c7(\theta_r-8\alpha) \\ s(7\theta_r) & s7(\theta_r-\alpha) & s7(\theta_r-2\alpha) & s7(\theta_r-3\alpha) & s7(\theta_r-4\alpha) & s7(\theta_r-5\alpha) & s7(\theta_r-6\alpha) & s7(\theta_r-7\alpha) & s7(\theta_r-8\alpha) \\ 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix} \times \tag{3.16}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \\ V_g \\ V_h \\ V_i \end{bmatrix} = \begin{bmatrix} V_{q1r} \\ V_{d1r} \\ V_{q3r} \\ V_{d3r} \\ V_{q5r} \\ V_{d5r} \\ V_{q7r} \\ V_{d7r} \\ V_{or} \end{bmatrix}$$

In equation (3.16) the derivative part of the transformation matrix can be written as:

$$T(\theta_r)\,pT(\theta_r)^{-1} =$$

$$\frac{2}{9}\begin{bmatrix} c(\theta) & c(\theta-\alpha) & c(\theta-2\alpha) & c(\theta-3\alpha) & c(\theta-4\alpha) & c(\theta-5\alpha) & c(\theta-6\alpha) & c(\theta-7\alpha) & c(\theta-8\alpha) \\ s(\theta) & s(\theta-\alpha) & s(\theta-2\alpha) & s(\theta-3\alpha) & s(\theta-4\alpha) & s(\theta-5\alpha) & s(\theta-6\alpha) & s(\theta-7\alpha) & s(\theta-8\alpha) \\ c(3\theta) & c3(\theta-\alpha) & c3(\theta-2\alpha) & c3(\theta-3\alpha) & c3(\theta-4\alpha) & c3(\theta-5\alpha) & c3(\theta-6\alpha) & c3(\theta-7\alpha) & c3(\theta-8\alpha) \\ s(3\theta) & s3(\theta-\alpha) & s3(\theta-2\alpha) & s3(\theta-3\alpha) & s3(\theta-4\alpha) & s3(\theta-5\alpha) & s3(\theta-6\alpha) & s3(\theta-7\alpha) & s3(\theta-8\alpha) \\ c(5\theta) & c5(\theta-\alpha) & c5(\theta-2\alpha) & c5(\theta-3\alpha) & c5(\theta-4\alpha) & c5(\theta-5\alpha) & c5(\theta-6\alpha) & c5(\theta-7\alpha) & c5(\theta-8\alpha) \\ s(5\theta) & s5(\theta-\alpha) & s5(\theta-2\alpha) & s5(\theta-3\alpha) & s5(\theta-4\alpha) & s5(\theta-5\alpha) & s5(\theta-6\alpha) & s5(\theta-7\alpha) & s5(\theta-8\alpha) \\ c(7\theta) & c7(\theta-\alpha) & c7(\theta-2\alpha) & c7(\theta-3\alpha) & c7(\theta-4\alpha) & c7(\theta-5\alpha) & c7(\theta-6\alpha) & c7(\theta-7\alpha) & c7(\theta-8\alpha) \\ s(7\theta) & s7(\theta-\alpha) & s7(\theta-2\alpha) & s7(\theta-3\alpha) & s7(\theta-4\alpha) & s7(\theta-5\alpha) & s7(\theta-6\alpha) & s7(\theta-7\alpha) & s7(\theta-8\alpha) \\ 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix} \times \frac{2}{9}$$

$$\omega_r \begin{bmatrix} -s(\theta) & -s(\theta-\alpha) & -s(\theta-2\alpha) & -s(\theta-3\alpha) & -s(\theta-4\alpha) & -s(\theta-5\alpha) & -s(\theta-6\alpha) & -s(\theta-7\alpha) & -s(\theta-8\alpha) \\ c(\theta) & c(\theta-\alpha) & c(\theta-2\alpha) & c(\theta-3\alpha) & c(\theta-4\alpha) & c(\theta-5\alpha) & c(\theta-6\alpha) & c(\theta-7\alpha) & c(\theta-8\alpha) \\ -3s(3\theta) & -3s3(\theta-\alpha) & -3s3(\theta-2\alpha) & -3s3(\theta-3\alpha) & -3s3(\theta-4\alpha) & -3s3(\theta-5\alpha) & -3s3(\theta-6\alpha) & -3s3(\theta-7\alpha) & -3s3(\theta-8\alpha) \\ 3c(3\theta) & 3c3(\theta-\alpha) & 3c3(\theta-2\alpha) & 3c3(\theta-3\alpha) & 3c3(\theta-4\alpha) & 3c3(\theta-5\alpha) & 3c3(\theta-6\alpha) & 3c3(\theta-7\alpha) & 3c3(\theta-8\alpha) \\ -5s(5\theta) & -5s5(\theta-\alpha) & -5s5(\theta-2\alpha) & -5s5(\theta-3\alpha) & -5s5(\theta-4\alpha) & -5s5(\theta-5\alpha) & -5s5(\theta-6\alpha) & -5s5(\theta-7\alpha) & -5s5(\theta-8\alpha) \\ 5c(5\theta) & 5c5(\theta-\alpha) & 5c5(\theta-2\alpha) & 5c5(\theta-3\alpha) & 5c5(\theta-4\alpha) & 5c5(\theta-5\alpha) & 5c5(\theta-6\alpha) & 5c5(\theta-7\alpha) & 5c5(\theta-8\alpha) \\ -7s(7\theta) & -7s7(\theta-\alpha) & -7s7(\theta-2\alpha) & -7s7(\theta-3\alpha) & -7s7(\theta-4\alpha) & -7s7(\theta-5\alpha) & -7s7(\theta-6\alpha) & -7s7(\theta-7\alpha) & -7s7(\theta-8\alpha) \\ 7c(7\theta) & 7c7(\theta-\alpha) & 7c7(\theta-2\alpha) & 7c7(\theta-3\alpha) & 7c7(\theta-4\alpha) & 7c7(\theta-5\alpha) & 7c7(\theta-6\alpha) & 7c7(\theta-7\alpha) & 7c7(\theta-8\alpha) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (3.17)$$

$$= \omega_r \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Substituting the phase currents by their equivalent values in the rotor reference frame from equation (3.11) results in:

$$\lambda_{qdor} = T(\theta_r) L_{ss} T^{-1}(\theta_r) i_{qdor} + T(\theta_r) \lambda_{abcdefghi\_pm} \tag{3.18}$$

The flux linkage due to the permanent magnet of the machine in the stator phases can be transformed to the rotor reference frame as equation (3.19). In this equation the resulting fluxes in the rotor reference frame are the first, third, fifth and the seventh sequences of the permanent magnets of the rotor.

$$T(\theta_r)\lambda_{abcdefghi\_pm} =$$

$$\frac{2}{9}\begin{bmatrix} c(\theta_r) & c(\theta_r-\alpha) & c(\theta_r-2\alpha) & c(\theta_r-3\alpha) & c(\theta_r-4\alpha) & c(\theta_r-5\alpha) & c(\theta_r-6\alpha) & c(\theta_r-7\alpha) & c(\theta_r-8\alpha) \\ s(\theta_r) & s(\theta_r-\alpha) & s(\theta_r-2\alpha) & s(\theta_r-3\alpha) & s(\theta_r-4\alpha) & s(\theta_r-5\alpha) & s(\theta_r-6\alpha) & s(\theta_r-7\alpha) & s(\theta_r-8\alpha) \\ c(3\theta_r) & c3(\theta_r-\alpha) & c3(\theta_r-2\alpha) & c3(\theta_r-3\alpha) & c3(\theta_r-4\alpha) & c3(\theta_r-5\alpha) & c3(\theta_r-6\alpha) & c3(\theta_r-7\alpha) & c3(\theta_r-8\alpha) \\ s(3\theta_r) & s3(\theta_r-\alpha) & s3(\theta_r-2\alpha) & s3(\theta_r-3\alpha) & s3(\theta_r-4\alpha) & s3(\theta_r-5\alpha) & s3(\theta_r-6\alpha) & s3(\theta_r-7\alpha) & s3(\theta_r-8\alpha) \\ c(5\theta_r) & c5(\theta_r-\alpha) & c5(\theta_r-2\alpha) & c5(\theta_r-3\alpha) & c5(\theta_r-4\alpha) & c5(\theta_r-5\alpha) & c5(\theta_r-6\alpha) & c5(\theta_r-7\alpha) & c5(\theta_r-8\alpha) \\ s(5\theta_r) & s5(\theta_r-\alpha) & s5(\theta_r-2\alpha) & s5(\theta_r-3\alpha) & s5(\theta_r-4\alpha) & s5(\theta_r-5\alpha) & s5(\theta_r-6\alpha) & s5(\theta_r-7\alpha) & s5(\theta_r-8\alpha) \\ c(7\theta_r) & c7(\theta_r-\alpha) & c7(\theta_r-2\alpha) & c7(\theta_r-3\alpha) & c7(\theta_r-4\alpha) & c7(\theta_r-5\alpha) & c7(\theta_r-6\alpha) & c7(\theta_r-7\alpha) & c7(\theta_r-8\alpha) \\ s(7\theta_r) & s7(\theta_r-\alpha) & s7(\theta_r-2\alpha) & s7(\theta_r-3\alpha) & s7(\theta_r-4\alpha) & s7(\theta_r-5\alpha) & s7(\theta_r-6\alpha) & s7(\theta_r-7\alpha) & s7(\theta_r-8\alpha) \\ 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix}\times$$

$$\begin{bmatrix} \lambda_{pma} \\ \lambda_{pmb} \\ \lambda_{pmc} \\ \lambda_{pmd} \\ \lambda_{pme} \\ \lambda_{pmf} \\ \lambda_{pmg} \\ \lambda_{pmh} \\ \lambda_{pmi} \end{bmatrix} = \begin{bmatrix} \lambda_{pmq1r} \\ \lambda_{pmd1r} \\ \lambda_{pmq3r} \\ \lambda_{pmd3r} \\ \lambda_{pmq5r} \\ \lambda_{pmd5r} \\ \lambda_{pmq7r} \\ \lambda_{pmd7r} \\ \lambda_{pm0r} \end{bmatrix} \tag{3.19}$$

The equation (3.18) also has a term which includes 'L_{SS}' representing the inductances of the stator of the machine. The inductances can be transformed to the rotor reference frame as equation (3.20).

$$T(\theta_r)L_{ss}T^{-1}(\theta_r)=$$

$$\frac{2}{9}\begin{bmatrix} c(\theta_r) & c(\theta_r-\alpha) & c(\theta_r-2\alpha) & c(\theta_r-3\alpha) & c(\theta_r-4\alpha) & c(\theta_r-5\alpha) & c(\theta_r-6\alpha) & c(\theta_r-7\alpha) & c(\theta_r-8\alpha) \\ s(\theta_r) & s(\theta_r-\alpha) & s(\theta_r-2\alpha) & s(\theta_r-3\alpha) & s(\theta_r-4\alpha) & s(\theta_r-5\alpha) & s(\theta_r-6\alpha) & s(\theta_r-7\alpha) & s(\theta_r-8\alpha) \\ c(3\theta_r) & c3(\theta_r-\alpha) & c3(\theta_r-2\alpha) & c3(\theta_r-3\alpha) & c3(\theta_r-4\alpha) & c3(\theta_r-5\alpha) & c3(\theta_r-6\alpha) & c3(\theta_r-7\alpha) & c3(\theta_r-8\alpha) \\ s(3\theta_r) & s3(\theta_r-\alpha) & s3(\theta_r-2\alpha) & s3(\theta_r-3\alpha) & s3(\theta_r-4\alpha) & s3(\theta_r-5\alpha) & s3(\theta_r-6\alpha) & s3(\theta_r-7\alpha) & s3(\theta_r-8\alpha) \\ c(5\theta_r) & c5(\theta_r-\alpha) & c5(\theta_r-2\alpha) & c5(\theta_r-3\alpha) & c5(\theta_r-4\alpha) & c5(\theta_r-5\alpha) & c5(\theta_r-6\alpha) & c5(\theta_r-7\alpha) & c5(\theta_r-8\alpha) \\ s(5\theta_r) & s5(\theta_r-\alpha) & s5(\theta_r-2\alpha) & s5(\theta_r-3\alpha) & s5(\theta_r-4\alpha) & s5(\theta_r-5\alpha) & s5(\theta_r-6\alpha) & s5(\theta_r-7\alpha) & s5(\theta_r-8\alpha) \\ c(7\theta_r) & c7(\theta_r-\alpha) & c7(\theta_r-2\alpha) & c7(\theta_r-3\alpha) & c7(\theta_r-4\alpha) & c7(\theta_r-5\alpha) & c7(\theta_r-6\alpha) & c7(\theta_r-7\alpha) & c7(\theta_r-8\alpha) \\ s(7\theta_r) & s7(\theta_r-\alpha) & s7(\theta_r-2\alpha) & s7(\theta_r-3\alpha) & s7(\theta_r-4\alpha) & s7(\theta_r-5\alpha) & s7(\theta_r-6\alpha) & s7(\theta_r-7\alpha) & s7(\theta_r-8\alpha) \\ 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix}\times$$

$$\begin{bmatrix} L_{ls}+L_{aa} & L_{ab} & L_{ac} & L_{ad} & L_{ae} & L_{af} & L_{ag} & L_{ah} & L_{ai} \\ L_{ba} & L_{ls}+L_{bb} & L_{bc} & L_{bd} & L_{be} & L_{bf} & L_{bg} & L_{bh} & L_{bi} \\ L_{ca} & L_{cb} & L_{ls}+L_{cc} & L_{cd} & L_{ce} & L_{cf} & L_{cg} & L_{ch} & L_{ci} \\ L_{da} & L_{db} & L_{dc} & L_{ls}+L_{dd} & L_{de} & L_{df} & L_{dg} & L_{dh} & L_{di} \\ L_{ea} & L_{eb} & L_{ec} & L_{ed} & L_{ls}+L_{ee} & L_{ef} & L_{eg} & L_{eh} & L_{ei} \\ L_{fa} & L_{fb} & L_{fc} & L_{fd} & L_{fe} & L_{ls}+L_{ff} & L_{fg} & L_{fh} & L_{fi} \\ L_{ga} & L_{gb} & L_{gc} & L_{gd} & L_{ge} & L_{gf} & L_{ls}+L_{gg} & L_{gh} & L_{gi} \\ L_{ha} & L_{hb} & L_{hc} & L_{hd} & L_{he} & L_{hf} & L_{hg} & L_{ls}+L_{hh} & L_{hi} \\ L_{ia} & L_{ib} & L_{ic} & L_{id} & L_{ie} & L_{if} & L_{ig} & L_{ih} & L_{ls}+L_{ii} \end{bmatrix}\times \tag{3.20}$$

$$\frac{2}{9}\begin{bmatrix} c(\theta_r) & c(\theta_r-\alpha) & c(\theta_r-2\alpha) & c(\theta_r-3\alpha) & c(\theta_r-4\alpha) & c(\theta_r-5\alpha) & c(\theta_r-6\alpha) & c(\theta_r-7\alpha) & c(\theta_r-8\alpha) \\ s(\theta_r) & s(\theta_r-\alpha) & s(\theta_r-2\alpha) & s(\theta_r-3\alpha) & s(\theta_r-4\alpha) & s(\theta_r-5\alpha) & s(\theta_r-6\alpha) & s(\theta_r-7\alpha) & s(\theta_r-8\alpha) \\ c(3\theta_r) & c3(\theta_r-\alpha) & c3(\theta_r-2\alpha) & c3(\theta_r-3\alpha) & c3(\theta_r-4\alpha) & c3(\theta_r-5\alpha) & c3(\theta_r-6\alpha) & c3(\theta_r-7\alpha) & c3(\theta_r-8\alpha) \\ s(3\theta_r) & s3(\theta_r-\alpha) & s3(\theta_r-2\alpha) & s3(\theta_r-3\alpha) & s3(\theta_r-4\alpha) & s3(\theta_r-5\alpha) & s3(\theta_r-6\alpha) & s3(\theta_r-7\alpha) & s3(\theta_r-8\alpha) \\ c(5\theta_r) & c5(\theta_r-\alpha) & c5(\theta_r-2\alpha) & c5(\theta_r-3\alpha) & c5(\theta_r-4\alpha) & c5(\theta_r-5\alpha) & c5(\theta_r-6\alpha) & c5(\theta_r-7\alpha) & c5(\theta_r-8\alpha) \\ s(5\theta_r) & s5(\theta_r-\alpha) & s5(\theta_r-2\alpha) & s5(\theta_r-3\alpha) & s5(\theta_r-4\alpha) & s5(\theta_r-5\alpha) & s5(\theta_r-6\alpha) & s5(\theta_r-7\alpha) & s5(\theta_r-8\alpha) \\ c(7\theta_r) & c7(\theta_r-\alpha) & c7(\theta_r-2\alpha) & c7(\theta_r-3\alpha) & c7(\theta_r-4\alpha) & c7(\theta_r-5\alpha) & c7(\theta_r-6\alpha) & c7(\theta_r-7\alpha) & c7(\theta_r-8\alpha) \\ s(7\theta_r) & s7(\theta_r-\alpha) & s7(\theta_r-2\alpha) & s7(\theta_r-3\alpha) & s7(\theta_r-4\alpha) & s7(\theta_r-5\alpha) & s7(\theta_r-6\alpha) & s7(\theta_r-7\alpha) & s7(\theta_r-8\alpha) \\ 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix}^{-1} = \begin{bmatrix} L_{q11} & L_{qd1} & L_{q1q3} & L_{q1d3} & L_{q1q5} & L_{q1d5} & L_{q1q7} & L_{q1d7} & L_{q10} \\ L_{qd1} & L_{d11} & L_{d1q3} & L_{d1d3} & L_{d1q5} & L_{d1d5} & L_{d1q7} & L_{d1d7} & L_{d10} \\ L_{q3q1} & L_{q3d1} & L_{q33} & L_{q3d3} & L_{q3q5} & L_{q3d5} & L_{q3q7} & L_{q3d7} & L_{q30} \\ L_{d3q1} & L_{d3d1} & L_{d3q3} & L_{d33} & L_{d3q5} & L_{d3d5} & L_{d3q7} & L_{d3d7} & L_{d30} \\ L_{q5q1} & L_{q5d1} & L_{q5q3} & L_{q5d3} & L_{q55} & L_{q5d5} & L_{q5d7} & L_{q5q7} & L_{q50} \\ L_{d5q1} & L_{d5d1} & L_{d5q3} & L_{d5d3} & L_{d5q5} & L_{d55} & L_{d5d7} & L_{d5q7} & L_{d50} \\ L_{q7q1} & L_{q7d1} & L_{q7q3} & L_{q7d3} & L_{q7q5} & L_{q7d5} & L_{q77} & L_{q7d7} & L_{q70} \\ L_{d7q1} & L_{d7d1} & L_{d7q3} & L_{d7d3} & L_{d7q5} & L_{d7d5} & L_{d7q7} & L_{d7d7} & L_{d70} \\ L_{0q1} & L_{0d1} & L_{0q3} & L_{0d3} & L_{0q5} & L_{0d5} & L_{0q7} & L_{0d7} & L_{00} \end{bmatrix}$$

Then using the permanent magnet flux linkages and the inductances of the machine in the rotor reference frame, the flux linkages of the machine for each harmonic can be represented in rotor reference frame as equation (3.21) [83].

$$
\begin{aligned}
\lambda_{q1r} &= L_{q11}i_{q1r} + L_{q1d1}i_{d1r} + L_{q1q3}i_{q3r} + L_{q1d3}i_{d3r} + L_{q1q5}i_{q5r} + L_{q1d5}i_{d5r} + L_{q1q7}i_{q7r} + L_{q1d7}i_{d7r} + L_{q10}i_{0} + \lambda_{pmq1r} \\
\lambda_{d1r} &= L_{d11}i_{d1r} + L_{d1q1}i_{q1r} + L_{d1d3}i_{d3r} + L_{d1q3}i_{q3r} + L_{d1d5}i_{d5r} + L_{d1q5}i_{q5r} + L_{d1d7}i_{d7r} + L_{d1q7}i_{q7r} + L_{d10}i_{0} + \lambda_{pmd1r} \\
\lambda_{q3r} &= L_{q3q1}i_{q1r} + L_{q3d1}i_{d1r} + L_{q33}i_{q3r} + L_{q3d3}i_{d3r} + L_{q3q5}i_{q5r} + L_{q3d5}i_{d5r} + L_{q3q7}i_{q7r} + L_{q3d7}i_{d7r} + L_{q30}i_{0} + \lambda_{pmq3r} \\
\lambda_{d3r} &= L_{d3d1}i_{d1r} + L_{d3d1}i_{d1r} + L_{d33}i_{d3r} + L_{d3q3}i_{q3r} + L_{d3d5}i_{d5r} + L_{d3q5}i_{q5r} + L_{d3d7}i_{d7r} + L_{d3q7}i_{q7r} + L_{d30}i_{0} + \lambda_{pmd3r} \\
\lambda_{q5r} &= L_{q5q1}i_{q1r} + L_{q5d1}i_{d1r} + L_{q5q3}i_{q3r} + L_{q5d3}i_{d3r} + L_{q55}i_{q5r} + L_{q5d5}i_{d5r} + L_{q5q7}i_{q7r} + L_{q5d7}i_{d7r} + L_{q50}i_{0} + \lambda_{pmq5r} \\
\lambda_{d5r} &= L_{d5d1}i_{d1r} + L_{d5q1}i_{q1r} + L_{d5d3}i_{d3r} + L_{d5q3}i_{q3r} + L_{d55}i_{d5r} + L_{d5q5}i_{q5r} + L_{d5d7}i_{d7r} + L_{d5q7}i_{q7r} + L_{d50}i_{0} + \lambda_{pmd5r} \\
\lambda_{q7r} &= L_{q7q1}i_{q1r} + L_{q7d1}i_{d1r} + L_{q7q3}i_{q3r} + L_{q7d3}i_{d3r} + L_{q7q5}i_{q5r} + L_{q7d5}i_{d5r} + L_{q77}i_{q7r} + L_{q7d7}i_{d7r} + L_{q70}i_{0} + \lambda_{pmq7r} \\
\lambda_{d7r} &= L_{d7d1}i_{d1r} + L_{d7q1}i_{q1r} + L_{d7d3}i_{d3r} + L_{d7q3}i_{q3r} + L_{d7d5}i_{d5r} + L_{d7q5}i_{q5r} + L_{d77}i_{d7r} + L_{d7q7}i_{q7r} + L_{d70}i_{0} + \lambda_{pmd7r} \\
\lambda_{0r} &= L_{0d1}i_{d1r} + L_{0q1}i_{q1r} + L_{0d3}i_{d3r} + L_{0q3}i_{q3r} + L_{0d5}i_{d5r} + L_{0q5}i_{q5r} + L_{0}i_{d7r} + L_{0q7}i_{q7r} + \lambda_{pmd0}
\end{aligned}
\tag{3.21}
$$

By separating the voltages of *q* and *d* axis of different components, the voltage equations of the machine could be represented as equation (3.22) [83].

$$
\begin{aligned}
V_{q1r} &= r_s i_{q1r} + \omega_r \lambda_{d1r} + p\lambda_{q1r} \\
V_{d1r} &= r_s i_{d1r} - \omega_r \lambda_{q1r} + p\lambda_{d1r} \\
V_{q3r} &= r_s i_{q3r} + 3\omega_r \lambda_{d3r} + p\lambda_{q3r} \\
V_{d3r} &= r_s i_{d3r} - 3\omega_r \lambda_{q3r} + p\lambda_{d3r} \\
V_{q5r} &= r_s i_{q5r} + 5\omega_r \lambda_{d5r} + p\lambda_{q5r} \\
V_{d5r} &= r_s i_{d5r} - 5\omega_r \lambda_{q5r} + p\lambda_{d5r} \\
V_{q7r} &= r_s i_{q7r} + 7\omega_r \lambda_{d7r} + p\lambda_{q7r} \\
V_{d7r} &= r_s i_{d7r} - 7\omega_r \lambda_{q7r} + p\lambda_{d7r} \\
V_{0r} &= r_s i_{0r} + p\lambda_{or}
\end{aligned} \tag{3.22}
$$

The generated torque of the machine can also be calculated using the co-energy equation. The co-energy of the machine can be presented as function of the stator currents and the flux linkages as equation (3.23) [152].

$$W_{co} = \frac{1}{2} I_s{}^t L_{ss} I_s + I_s{}^t \lambda_{pm} \tag{3.23}$$

Where:

$$I_s = [I_a \quad I_b \quad I_c \quad I_d \quad I_e \quad I_f \quad I_g \quad I_h \quad I_i]^t \tag{3.24}$$

From the co-energy equation, the electromagnetic torque can be derived as:

$$T_e = \frac{\partial W_{co}}{\partial \theta_{rm}} \tag{3.25}$$

The equation (3.25) is equal to the equation (3.26).

$$T_e = \frac{1}{2} I_s{}^t \frac{\partial L_{ss}}{\partial \theta_{rm}} I_s + I_s{}^t \frac{\partial \lambda_{pm}}{\partial \theta_{rm}} \tag{3.26}$$

Since the previous equations are in term of the electrical angle, the mechanical angle of the equation (3.26) needs to be converted to the electrical equation as equation (3.27).

$$\theta_r = \frac{P}{2} \theta_{rm} \tag{3.27}$$

Therefore, the torque equation changes to the equation (3.28).

$$T_e = \frac{P}{2} \frac{1}{2} I_s{}^t \frac{\partial L_{ss}}{\partial \theta_r} I_s + \frac{P}{2} I_s{}^t \frac{\partial \lambda_{pm}}{\partial \theta_r} \tag{3.28}$$

Substituting the stator currents with their corresponding values in rotor reference frame results in:

$$T_e = \frac{9}{2} \frac{P}{2} (I_{qdor})^t T(\theta_r) \frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} I_{qdor} + \frac{9}{2} \frac{P}{2} (I_{qdor})^t T(\theta_r) \frac{\partial \lambda_{pmr}}{\partial \theta_r} \tag{3.29}$$

The equation (3.29) can be rewritten as equation (3.30)

$$T_e = \frac{9}{2} \frac{P}{2} \begin{bmatrix} i_{q1r} \\ i_{d1r} \\ i_{q3r} \\ i_{d3r} \\ i_{q5r} \\ i_{d5r} \\ i_{q7r} \\ i_{d7r} \\ i_{0r} \end{bmatrix}^t T(\theta_r) \frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} \begin{bmatrix} i_{q1r} \\ i_{d1r} \\ i_{q3r} \\ i_{d3r} \\ i_{q5r} \\ i_{d5r} \\ i_{q7r} \\ i_{d7r} \\ i_{0r} \end{bmatrix} + \frac{9}{2} \frac{P}{2} \begin{bmatrix} i_{q1r} \\ i_{d1r} \\ i_{q3r} \\ i_{d3r} \\ i_{q5r} \\ i_{d5r} \\ i_{q7r} \\ i_{d7r} \\ i_{0r} \end{bmatrix}^t T(\theta_r) \frac{\partial \lambda_{pmr}}{\partial \theta_r} \tag{3.30}$$

Finally, the electromagnetic torque of the machine can be represented as:

$$T_e = \frac{9P}{4}\begin{bmatrix} (L_{q11} - L_{d11})i_{q1r}i_{d1r} + 3(L_{q33} - L_{d33})i_{q3r}i_{d3r} \\ + 5(L_{q55} - L_{d55})i_{q5r}i_{d5r} + 7(L_{q77} - L_{d77})i_{q7r}i_{d7r} + \\ (L_{q1q3} - 3L_{d3d1})i_{q3r}i_{d1r} + (L_{q1q5} - 5L_{d5d1})i_{q5r}i_{d1r} + \\ (L_{q1q7} - 7L_{d7d1})i_{q7r}i_{d1r} + (3L_{q3q1} - L_{d1d3})i_{d3r}i_{q1r} + \\ (3L_{q3q5} - 5L_{d5d3})i_{d3r}i_{q5r} + (3L_{q3q7} - 7L_{d7d3})i_{d3r}i_{q7r} + \\ (5L_{q5q3} - 3L_{d3d5})i_{d5r}i_{q3r} + (7L_{q7q3} - 3L_{d3d7})i_{d7r}i_{q3r} + \\ (5L_{q5q1} - L_{d1d5})i_{d5r}i_{q1r} + (7L_{q7q1} - L_{d1d7})i_{d7r}i_{q1r} + \\ (5L_{q5q7} - 7L_{d7d5})i_{d5r}i_{q7r} + (7L_{q7q5} - 5L_{d5d7})i_{d7r}i_{q5r} + \\ (i_{q1r}\lambda_{dm1r} + 3i_{q3r}\lambda_{dm3r} + 5i_{q5r}\lambda_{dm5r} + 7i_{q7r}\lambda_{dm7r}) + \\ (i_{d1r}\lambda_{qm1r} + 3i_{d3r}\lambda_{qm3r} + 5i_{d5r}\lambda_{qm5r} + 7i_{d7r}\lambda_{qm7r}) \end{bmatrix} \tag{3.31}$$

The dynamic equation governing rotor speed can also be derived using the electromagnetic and load torque as equation (3.32). In this equation, *'P'* is the number of poles of the machine, 'ω_r' is the rotor speed, '*B*' is the friction coefficient and 'T_L' is the mechanical load torque applied to the machine.

$$T_e = J\left(\frac{2}{P}\right)p\omega_r + T_L + B\omega_r \tag{3.32}$$

3.2.1 Generating the Parameters of the Nine Phase IPM

After deriving the model equations, the next step is to generate the essential parameters such as inductances and flux linkages of the machine. This step can start from the clock diagram of the 9 phase machine which is shown in the Figure 3.1 (b) [83]. As it can be seen the machine has four poles with full pitch, double layer and concentrated windings. The machine has 36 slots each slot covers 40 degrees (electrical angle) of the stator circumferential.

Using the same method as presented in [162], based on this clock diagram the turn function of the stator phases of the machine can be generated according to the Figures 3.2 to 3.4.

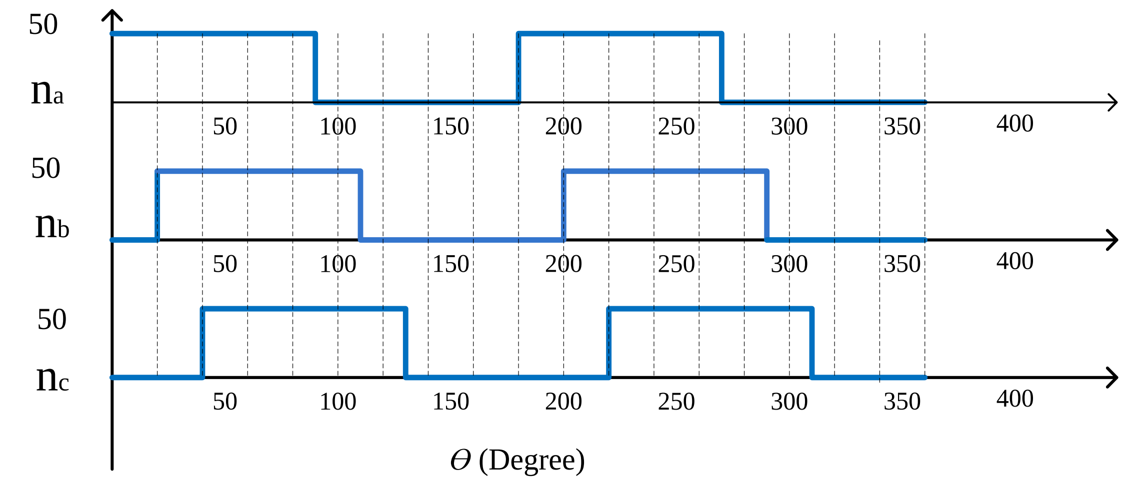


Figure 3.2: The turn functions of the phases a, b and c.

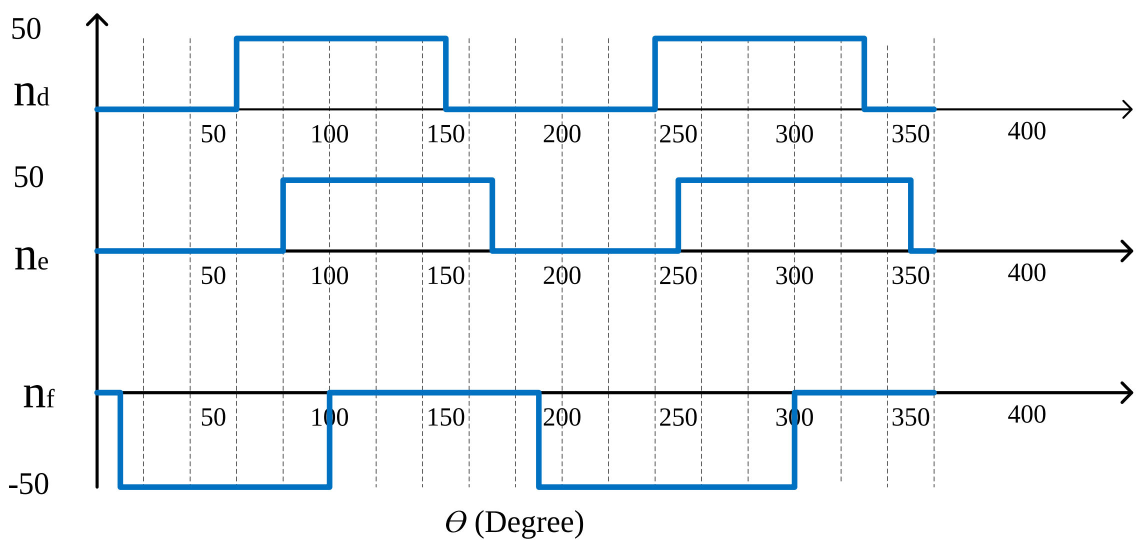


Figure 3.3: The turn functions of the phases d, e and f.

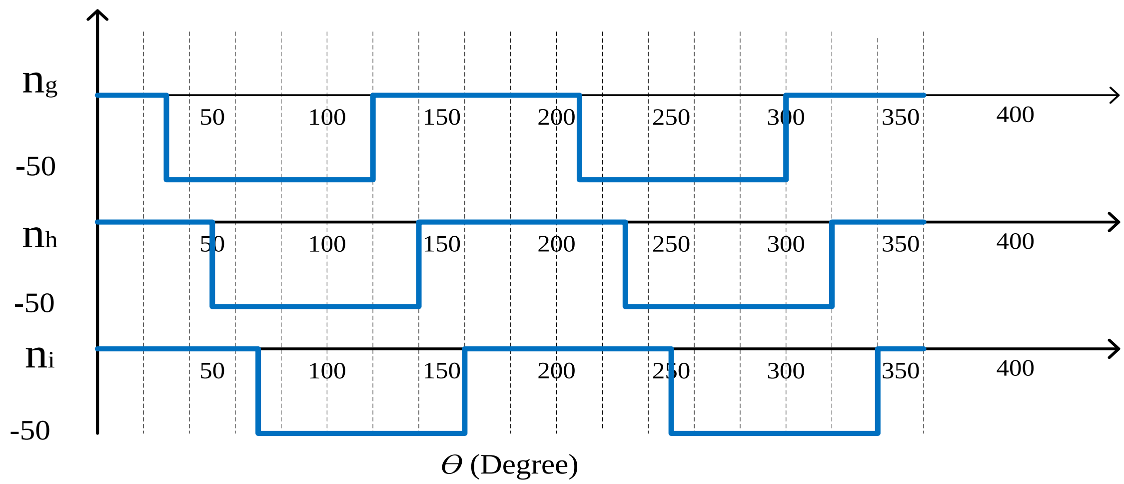


Figure 3.4: The turn functions of the phases g, h and i.

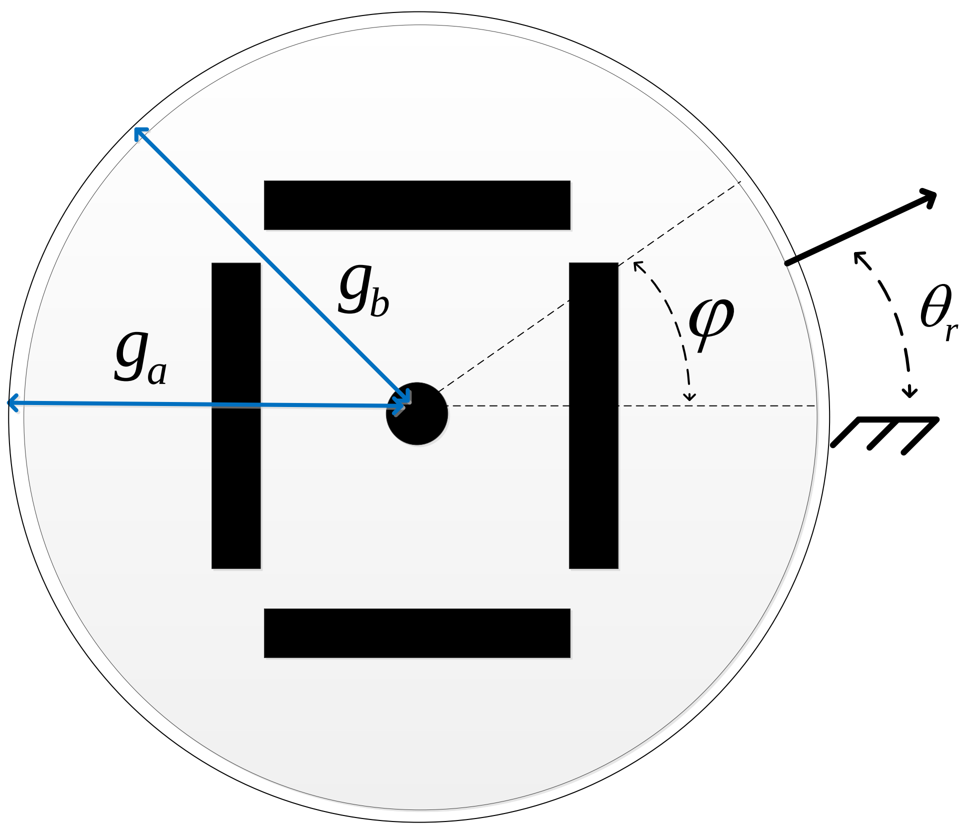


Figure 3.5: The rotor of the IPM machine-showing the effective “airgap” lengths.

The rotor of the machine is shown in the Figure 3.5. It can be seen that the rotor has four bars of the permanent magnet materials buried inside. The permanent magnets have lower permeability compared with the rotor iron. This fact causes different effective air gap lengths in 'g_a' and 'g_b' directions. Based on the Figure 3.5, the air gap function can be shown as Figure 3.6 [83].

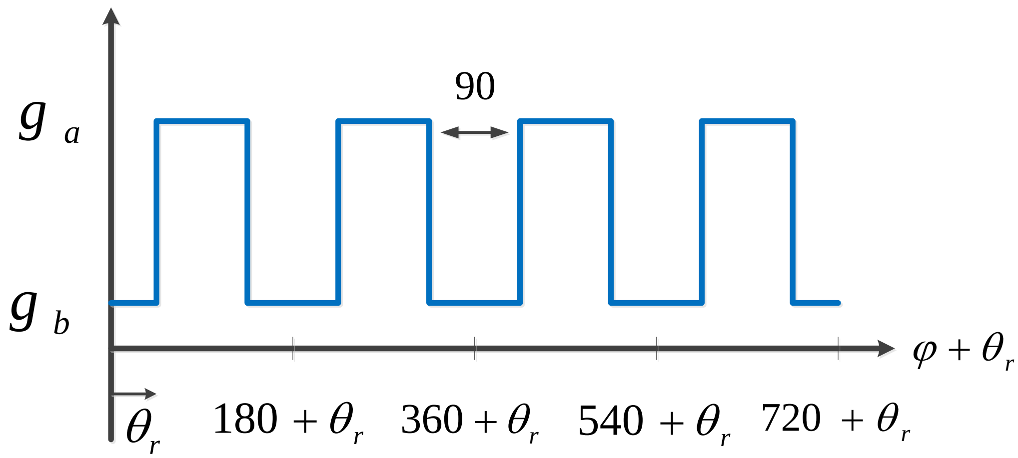
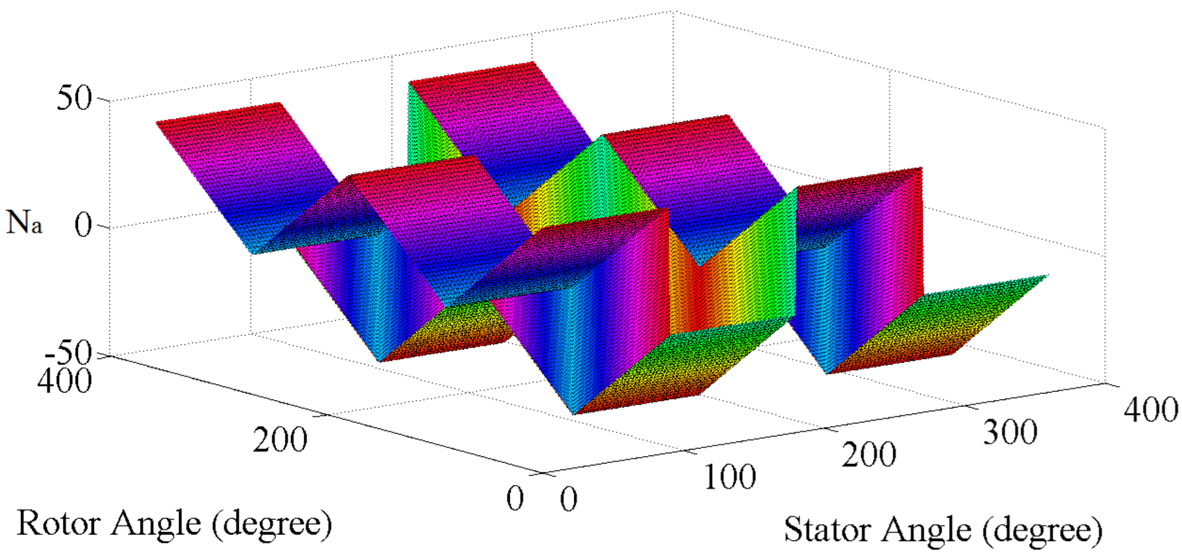


Figure 3.6: The air gap function.

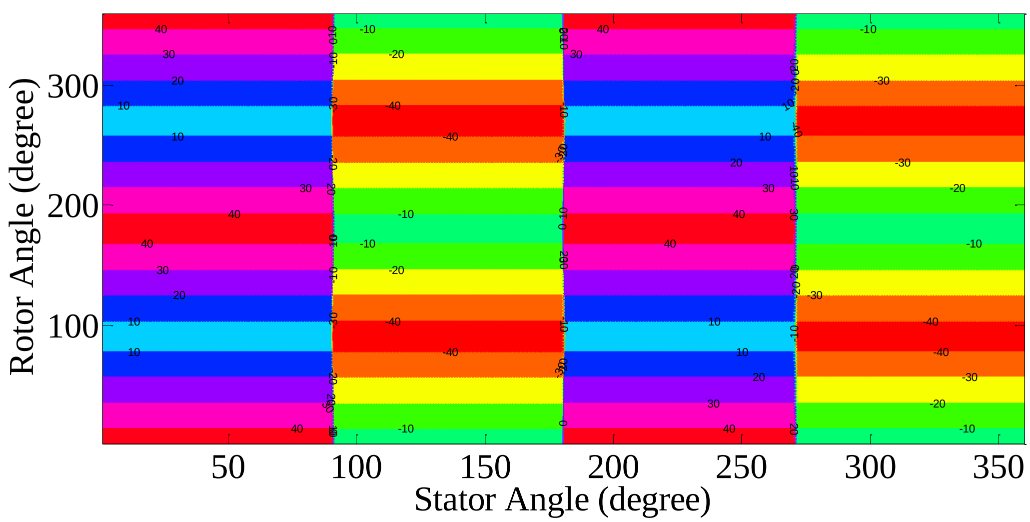
Using Figures 3.5 and 3.6 and the equation (3.33), the winding function of each phase can be calculated [153].

$$N_w(\theta) = n_w(\theta) - \frac{\int_0^{2\pi} \frac{n_w(\theta)}{g(\theta,\theta_r)} d\theta}{\int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} d\theta} \tag{3.33}$$

Using the equation (3.33) the winding functions of the phase stator phases are generated as a function of rotor and circumferential angle of the stator. Figure 3.7 (a) shows the winding function of the phase 'a'. Also the Figure 3.7 (b) shows contour of the same figure from the top view.



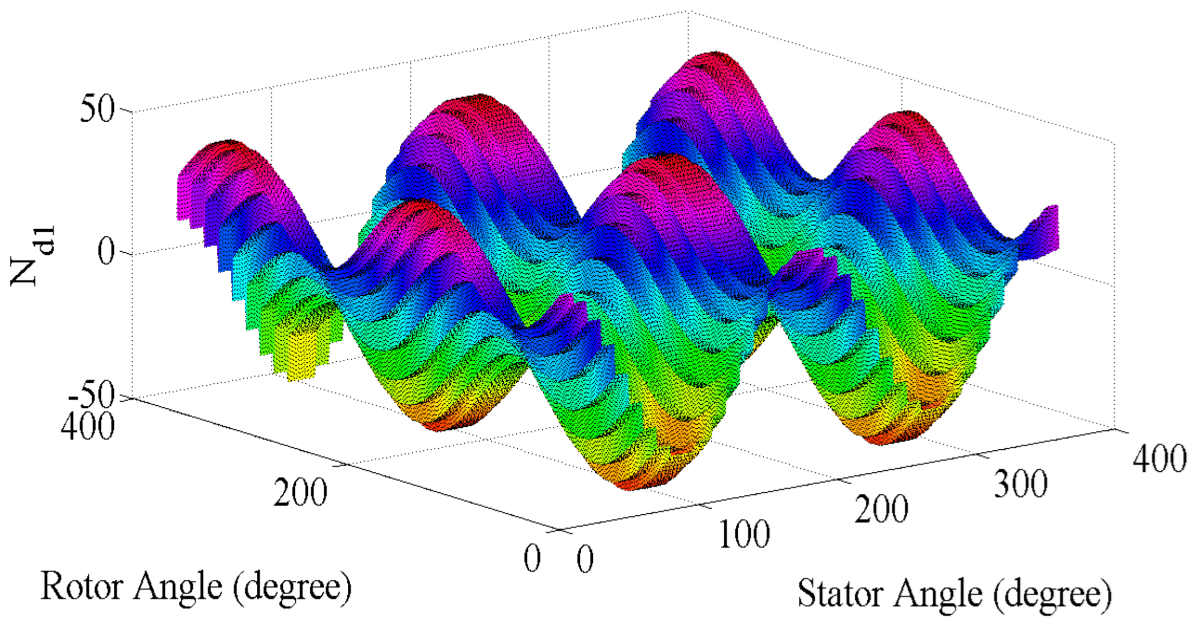
(a)



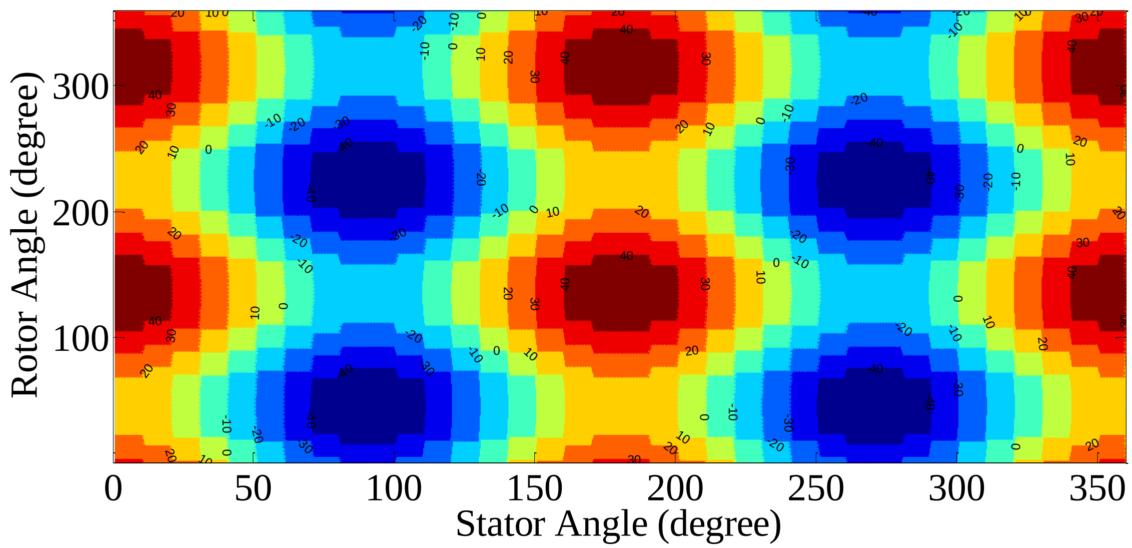
(b)

Figure 3.7: (a) The winding function phase 'a', (b) The contour from the top view.

Winding functions can be transformed to the stationary reference frame using the transformation matrix of equation (3.10) with '$\theta_r = 0$' to obtain the 'N_q' and 'N_d' for different sequences. The different sequences of the winding function can be used for deriving the equivalent circuit of the machine for different harmonics which will be used for the high frequency analysis. Figures 3.8 and 3.9 show the main components of the winding function for q and d axis in stationary reference frame. The higher order components of the winding functions are shown in Figures 3.10 to 3.16.

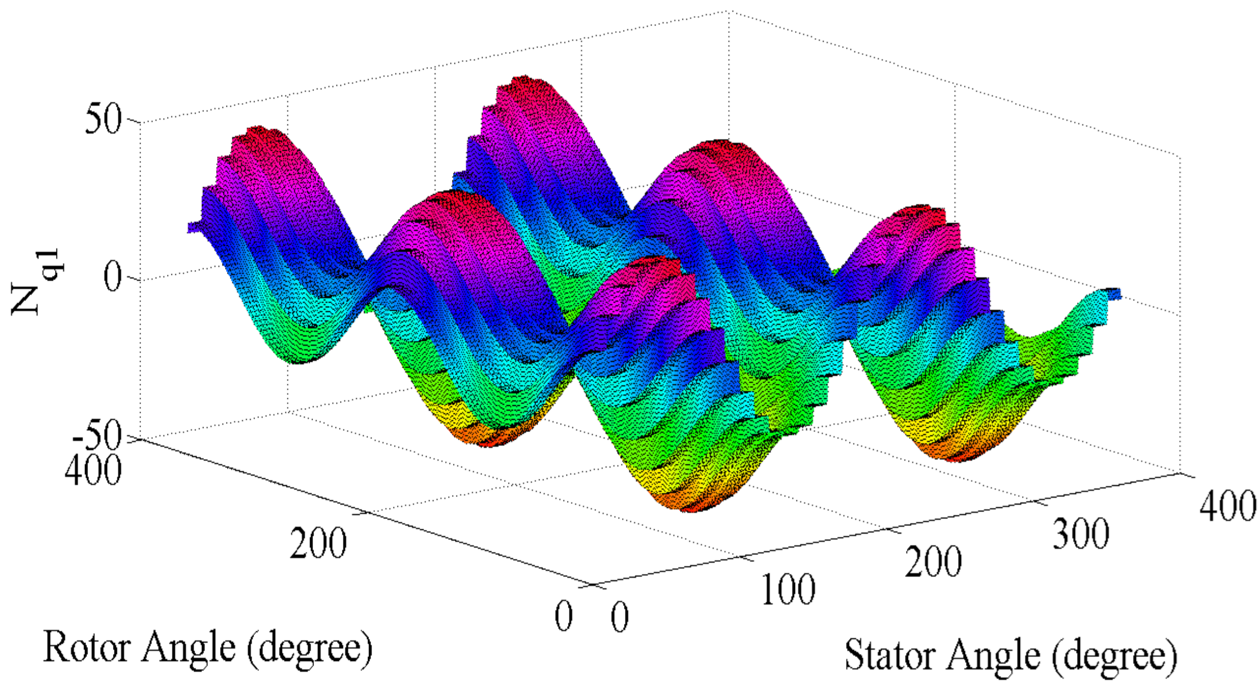


(a)

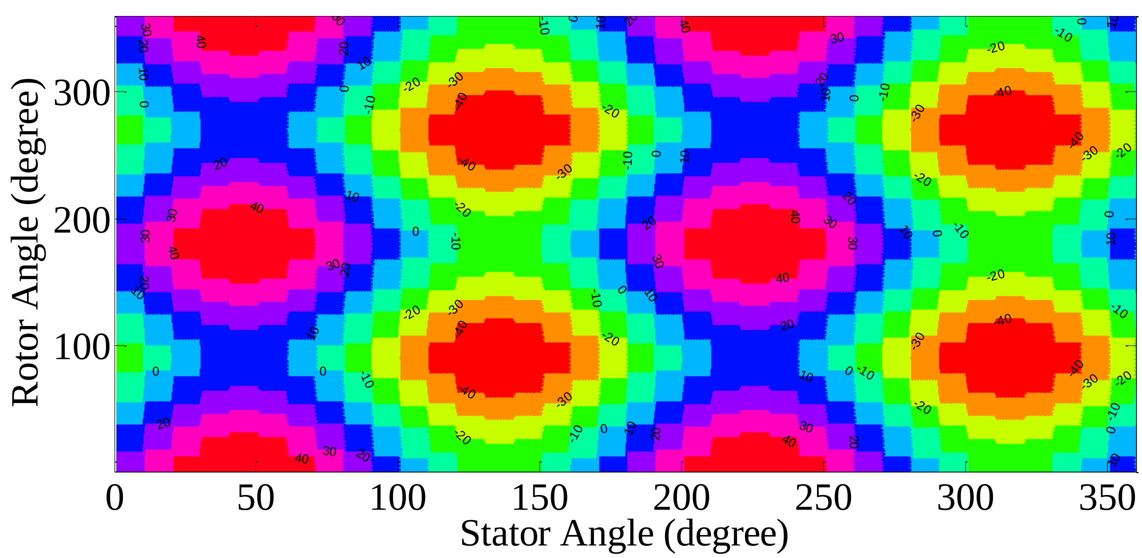


(b)

Figure 3.8: (a) The first sequence of stator winding function vs. stator and rotor angle in d axis of stationary reference frame, (b) The contour from the top view.



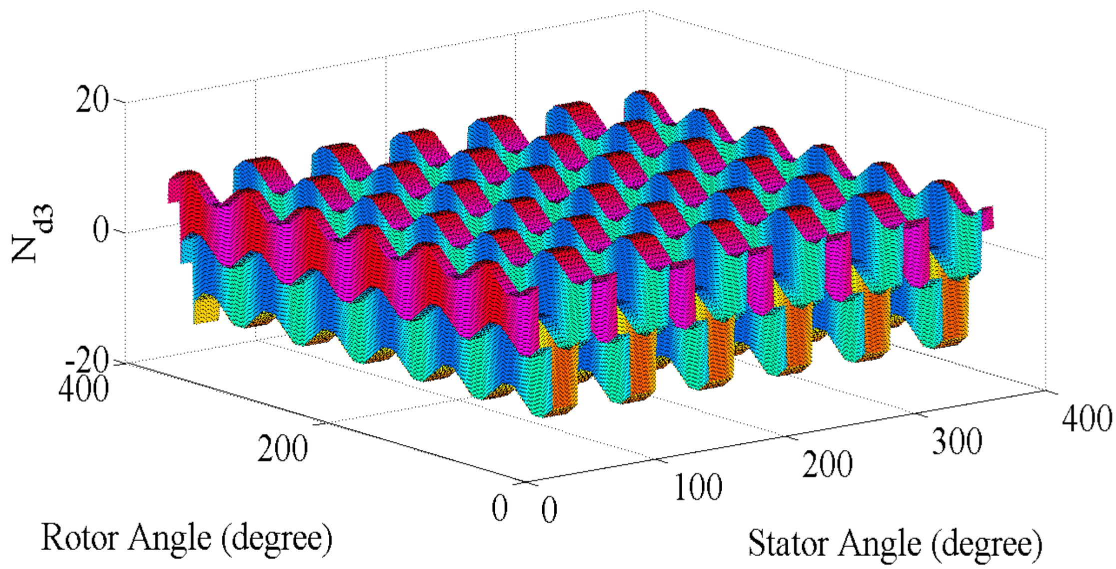
(a)



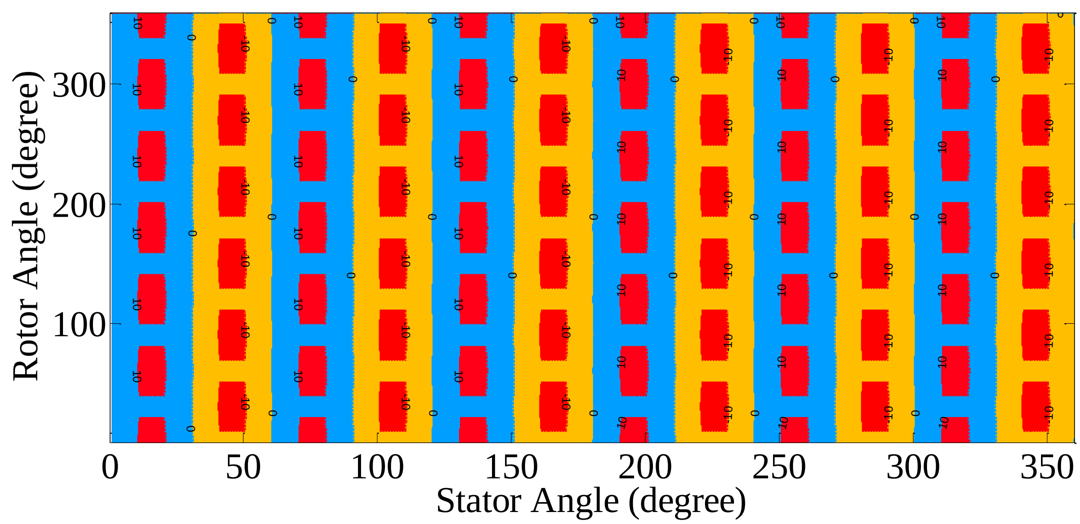
(b)

Figure 3.9: (a) The first sequence of stator winding function vs. stator and rotor angle in q axis of stationary reference frame, (b) The contour from the top view.

Figures 3.10 and 3.11 show the third component of the winding functions of the machine stator for d and q axis respectively. These components are the ones that will be used for high frequency injection analysis.



(a)



(b)

Figure 3.10: (a) The third sequence of stator winding function vs. stator and rotor angle in d axis of stationary reference frame, (b) The contour from the top view.

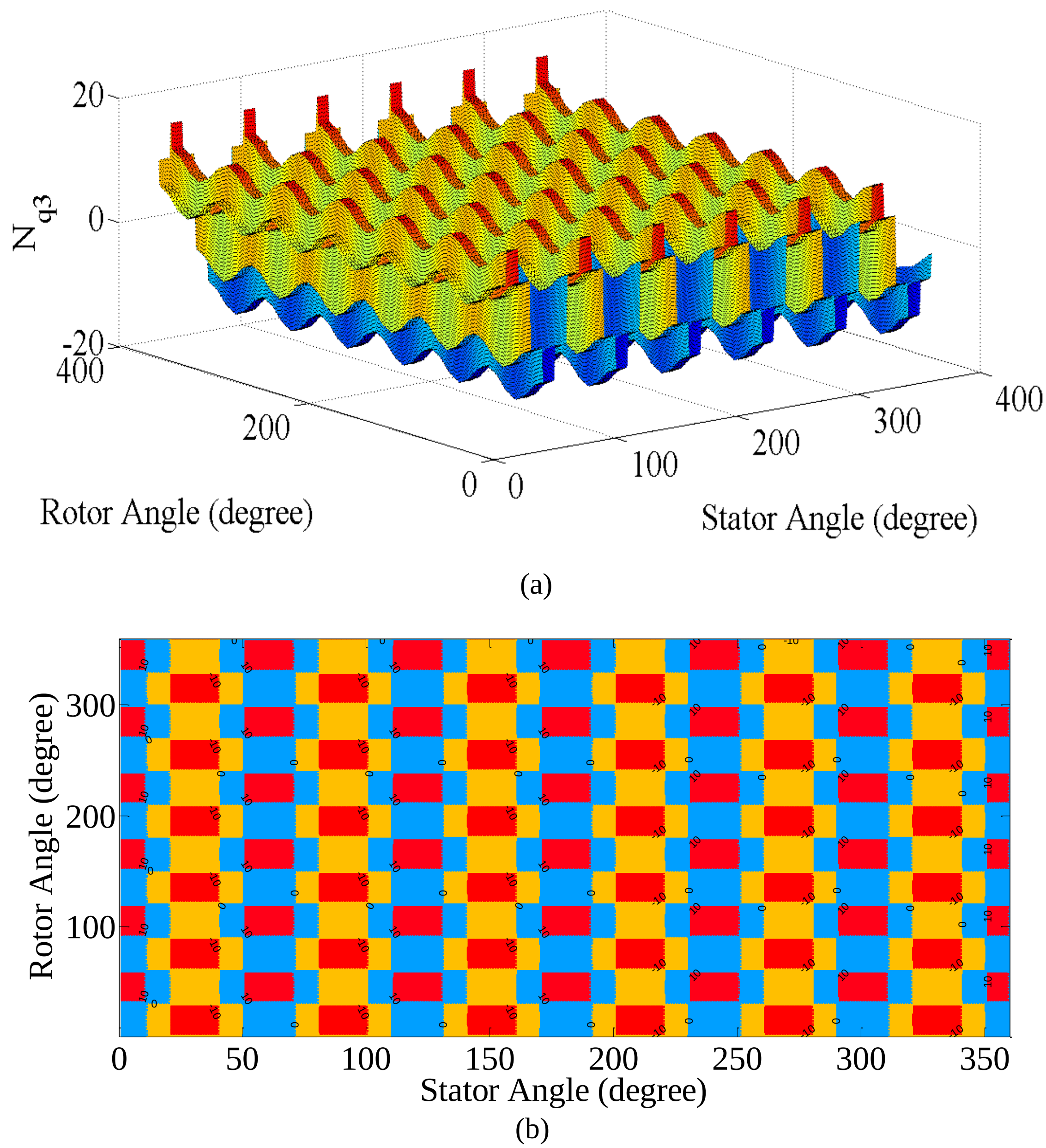


Figure 3.11: (a) The third sequence of stator winding function vs. stator and rotor angle in q axis of stationary reference frame, (b) The contour from the top view.

Figures 3.12 and 3.13 show the fifth components of the winding functions of the machine stator for d and q axis respectively.

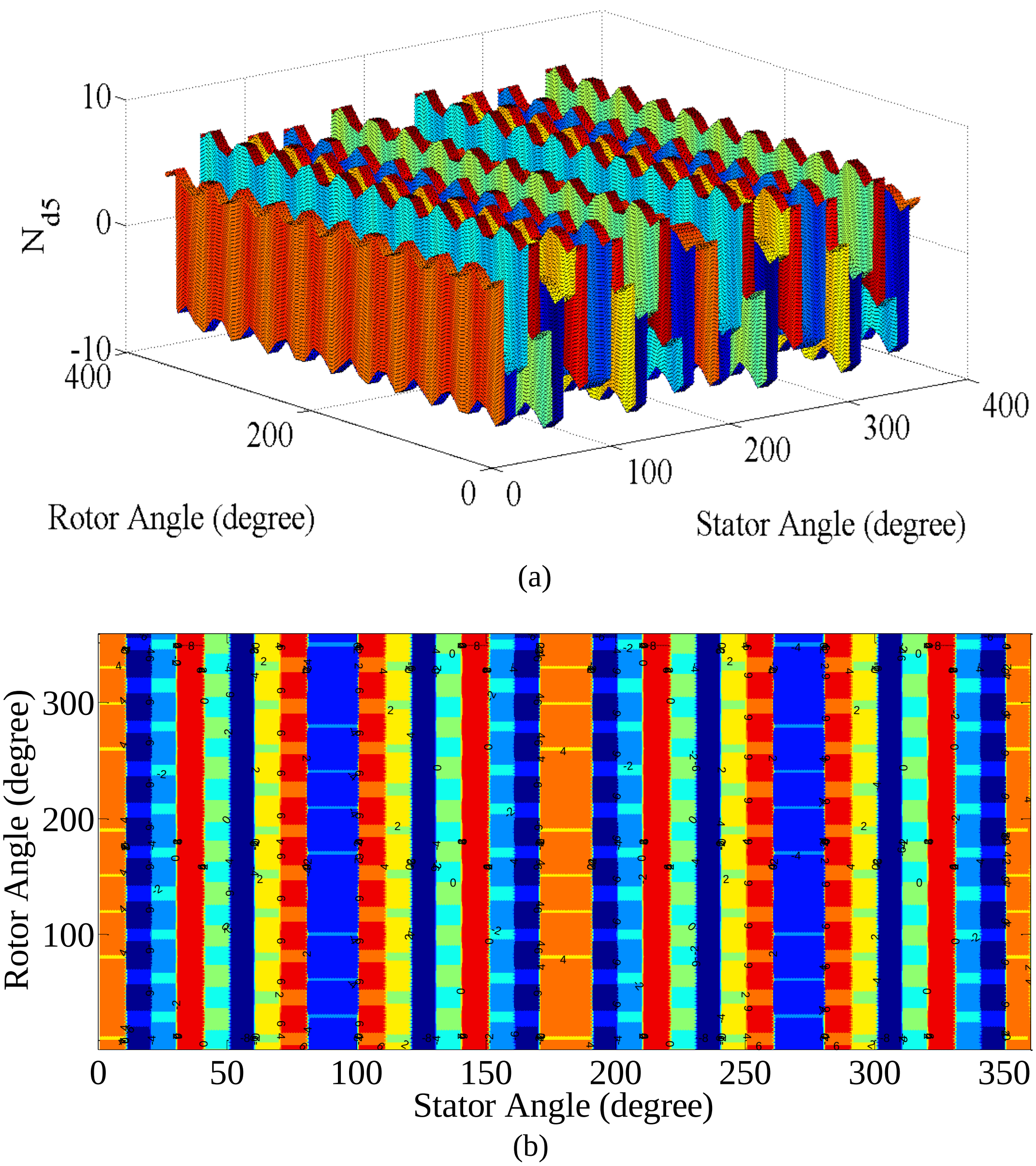
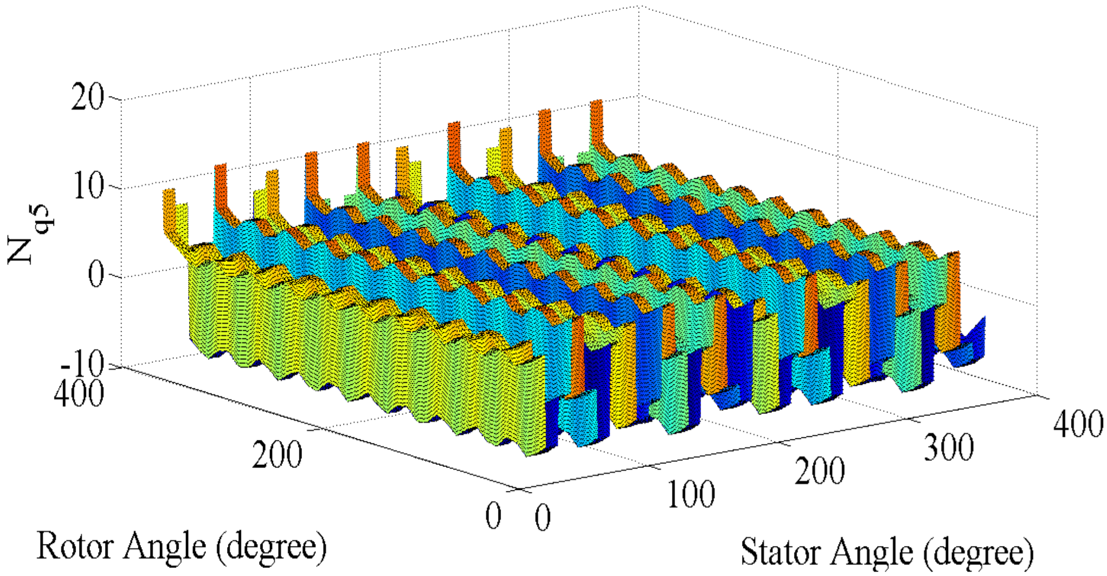
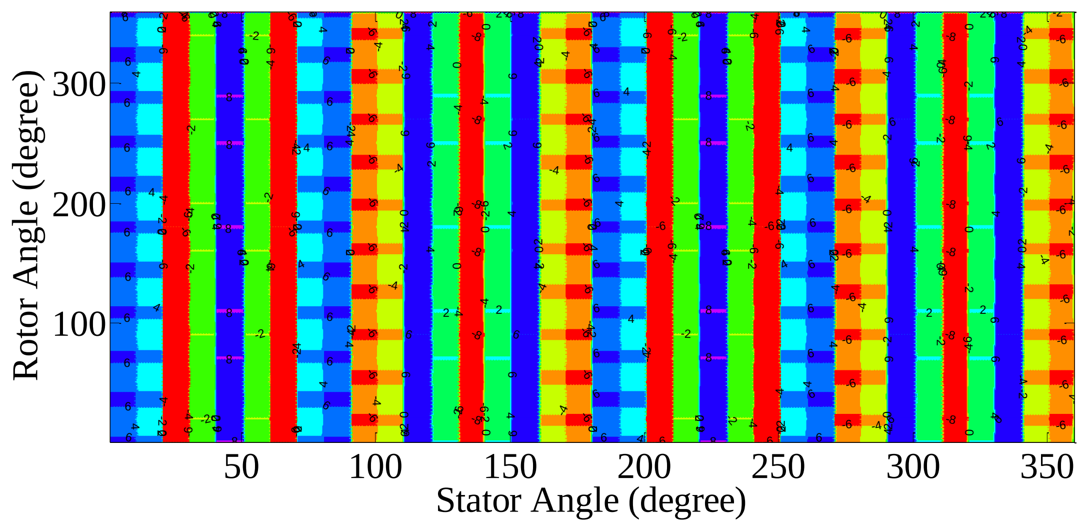


Figure 3.12: (a) The fifth sequence of stator winding function vs. stator and rotor angle in d axis of stationary reference frame, (b) The contour from the top view.



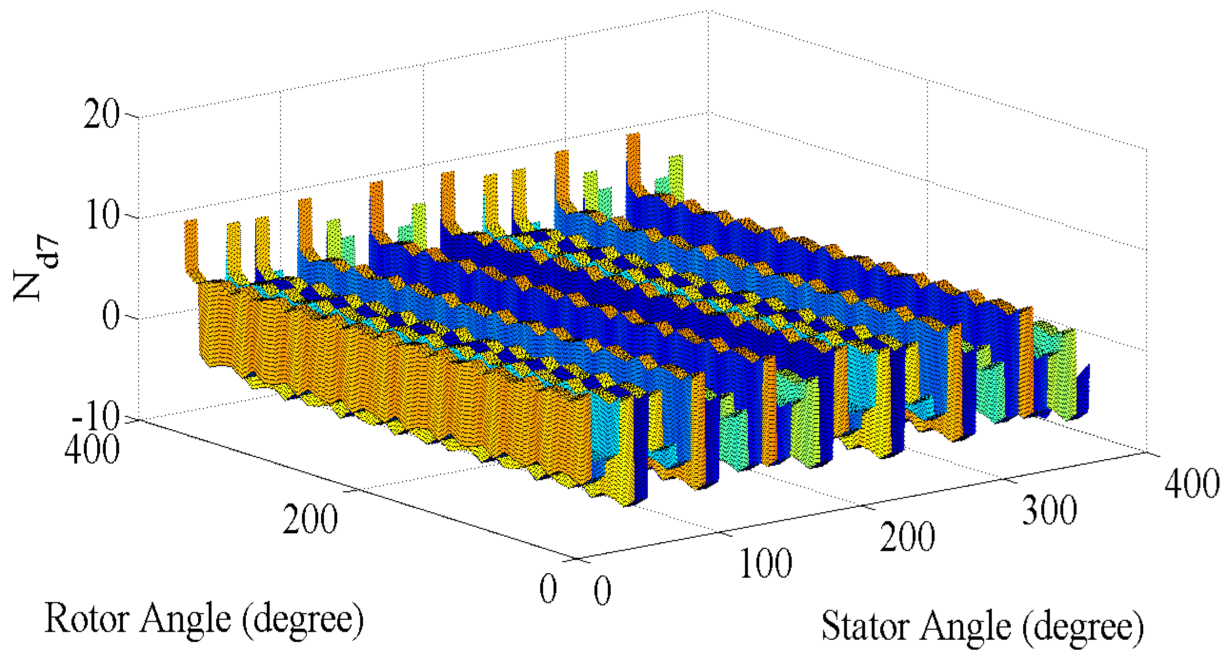
(a)



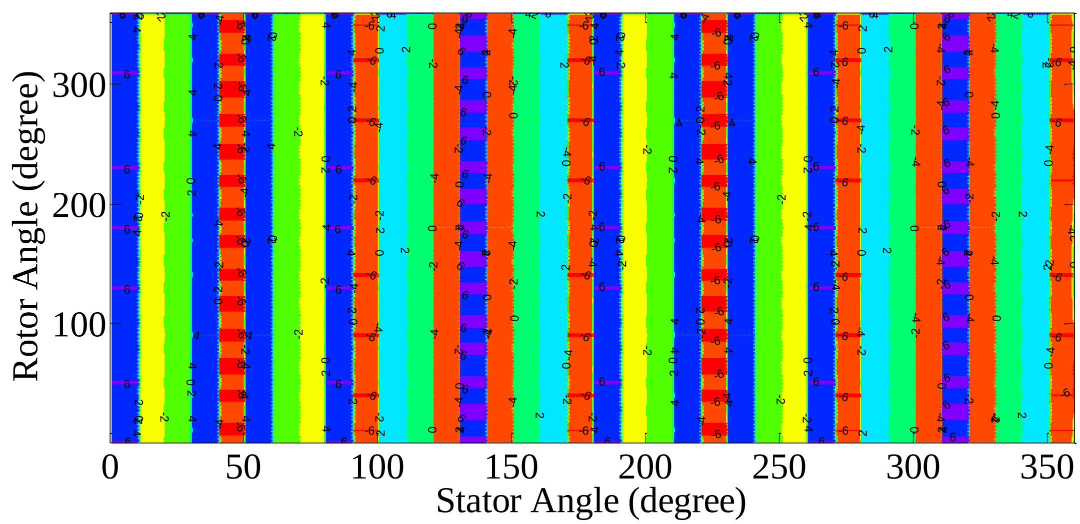
(b)

Figure 3.13: (a) The fifth sequence of stator winding function vs. stator and rotor angle in q axis of stationary reference frame, (b) The contour from the top view.

Figures 3.14 and 3.15 show the seventh components of the winding functions of the machine stator for d and q axis respectively.

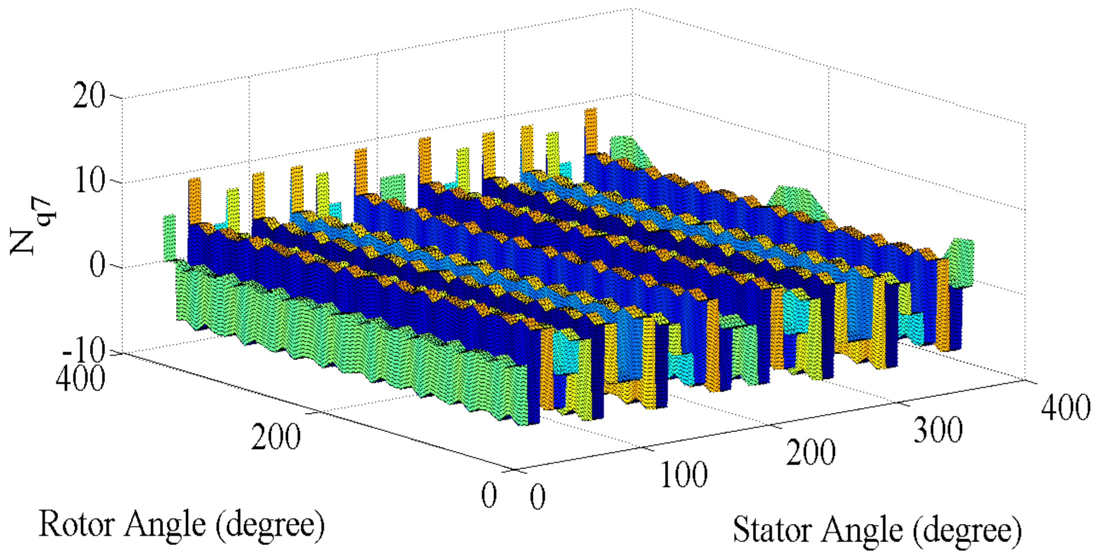


(a)

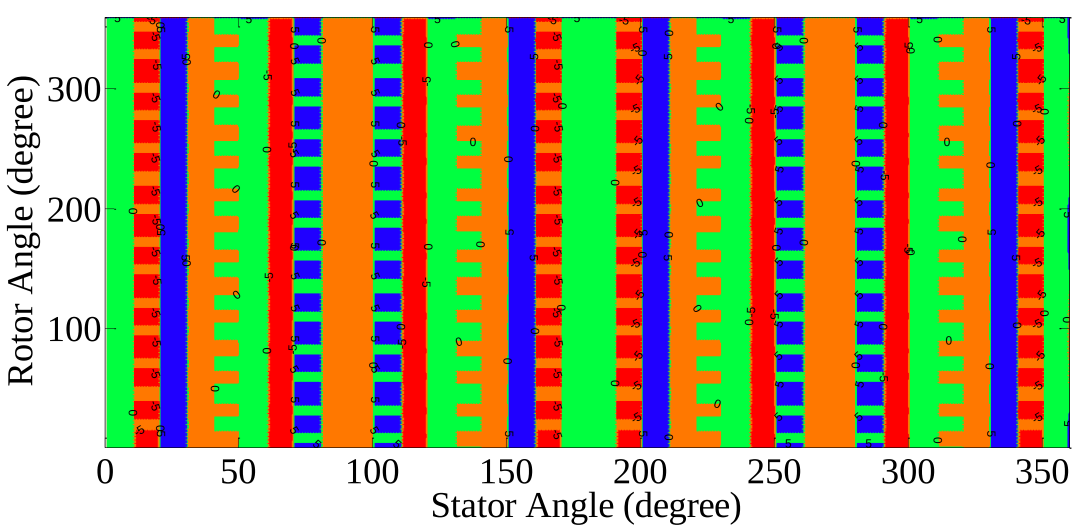


(b)

Figure 3.14: (a) The seventh sequence of stator winding function vs. stator and rotor angle in d axis of stationary reference frame, (b) The contour from the top view.

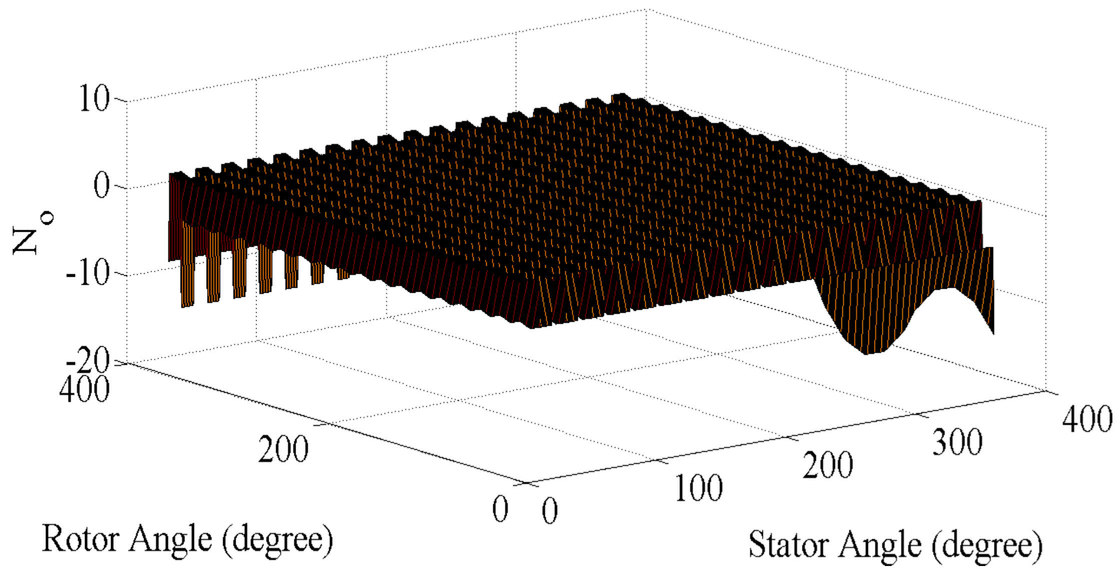


(a)

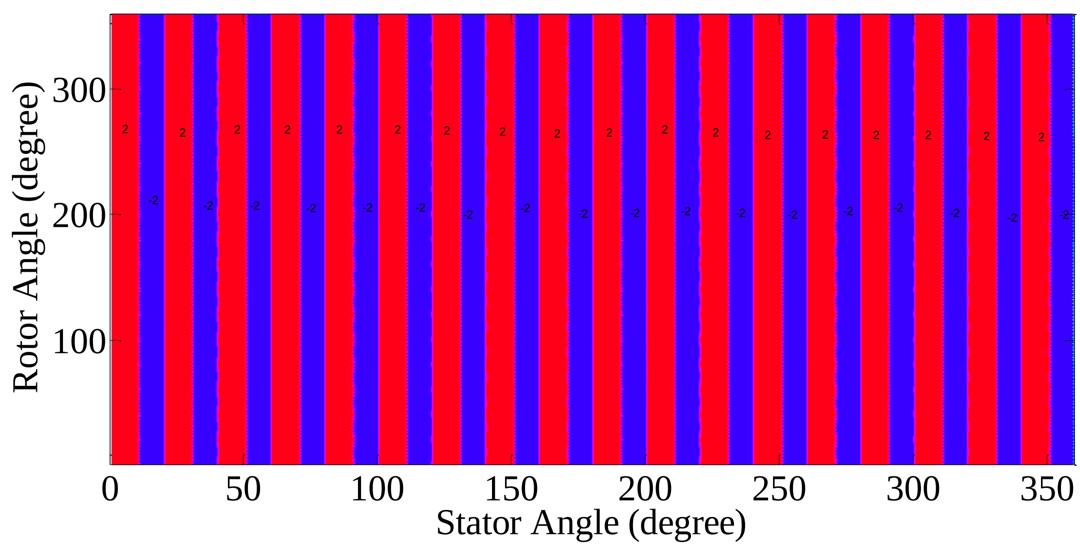


(b)

Figure 3.15: (a) The seventh sequence of stator winding function vs. stator and rotor angle in q axis of stationary reference frame, (b) The contour from the top view.



(a)



(b)

Figure 3.16: (a) The zero sequence of stator winding function vs. stator and rotor angle in stationary reference frame, (b) The contour from the top view.

Figure 3.16 shows zero sequence of the winding function in stationary reference frame. It can be seen that the N_q and N_d are changing with the changes of the rotor angle. Using the generated winding functions and equation (3.34) the self and mutual inductances of the machine phases can be calculated. Figures 3.17 to 3.25 show the mutual and self-inductances of the machine corresponding to each stator phase. In equation (3.34) 'r' is the radius of the rotor, 'l' is the machine length, 'μ_0' is the permeability of the machine iron, 'n_j' is the turn function of the phase 'j', 'N_k' is the winding function of the phase 'k' and '$g(\theta,\theta_r)$' is the air gap function of the machine [152].

$$L_{jk} = \mu_o r l \int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} n_j(\theta) N_k(\theta) d(\theta) \tag{3.34}$$

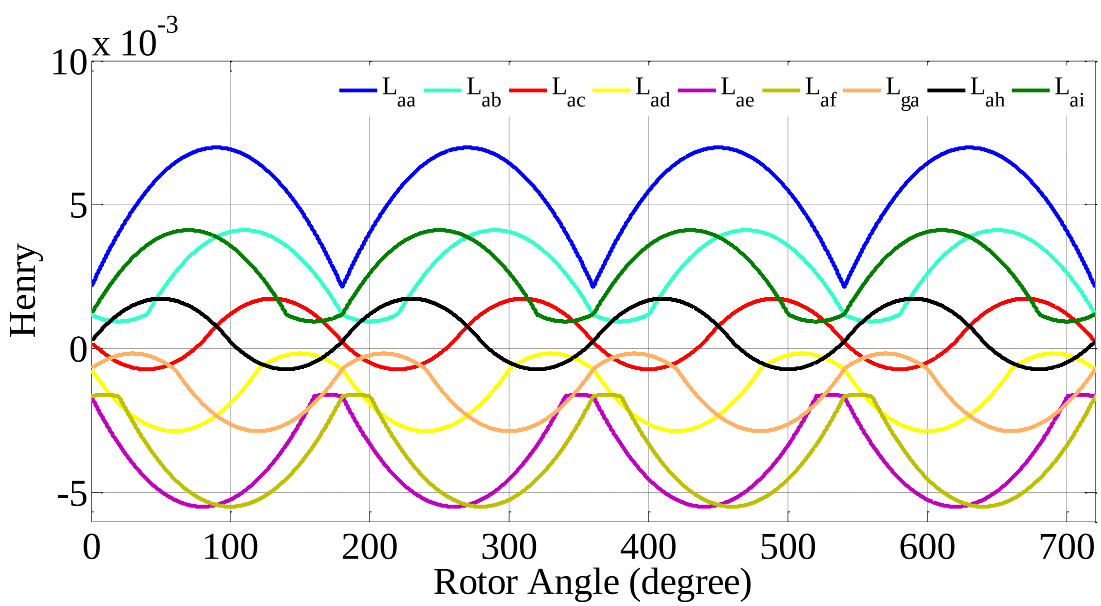


Figure 3.17: The self and mutual inductances corresponding to phase 'a'.

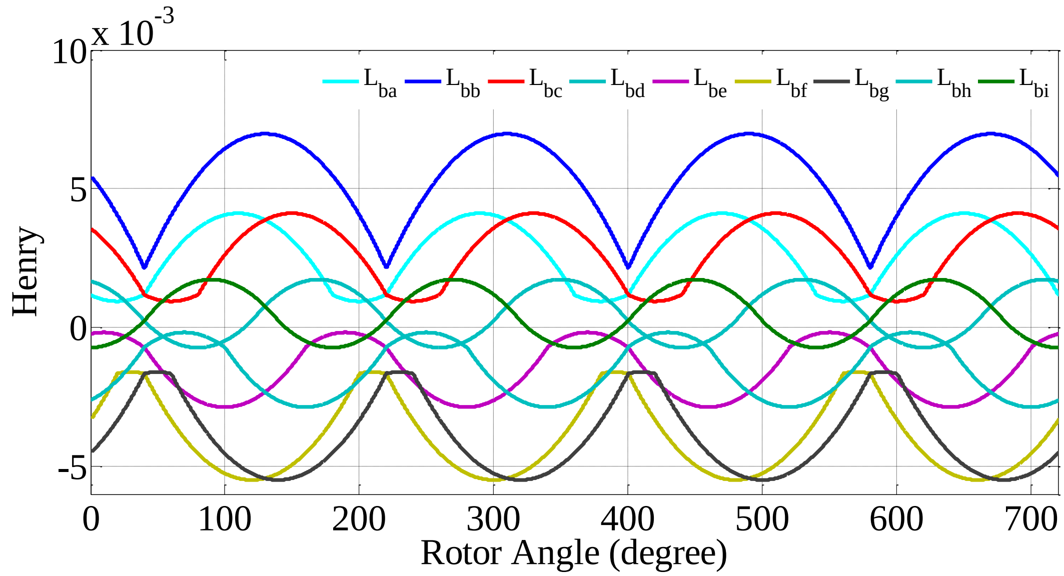


Figure 3.18: The self and mutual inductances corresponding to phase ‘b’.

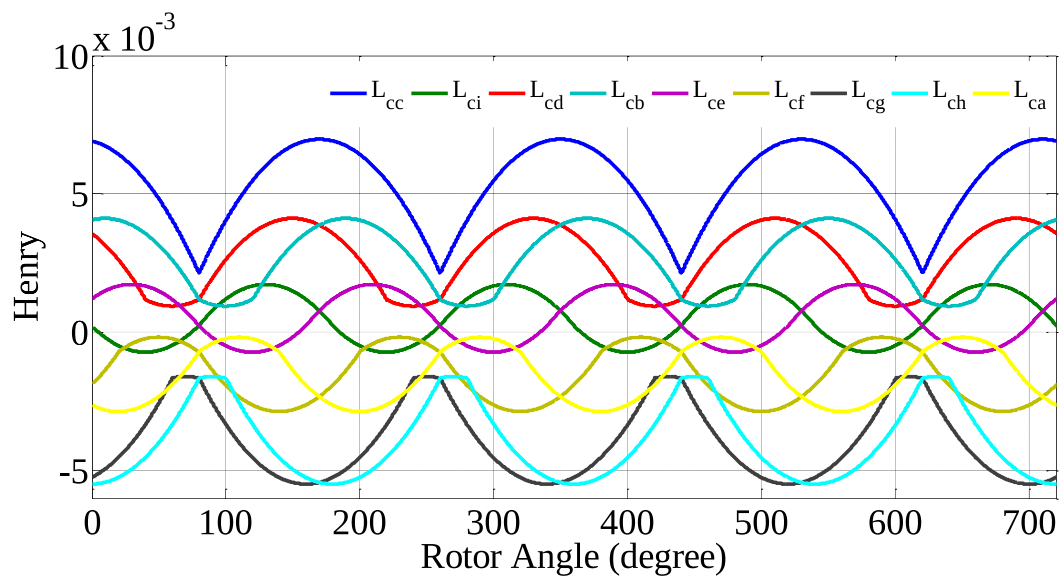


Figure 3.19: The self and mutual inductances corresponding to phase ‘c’.

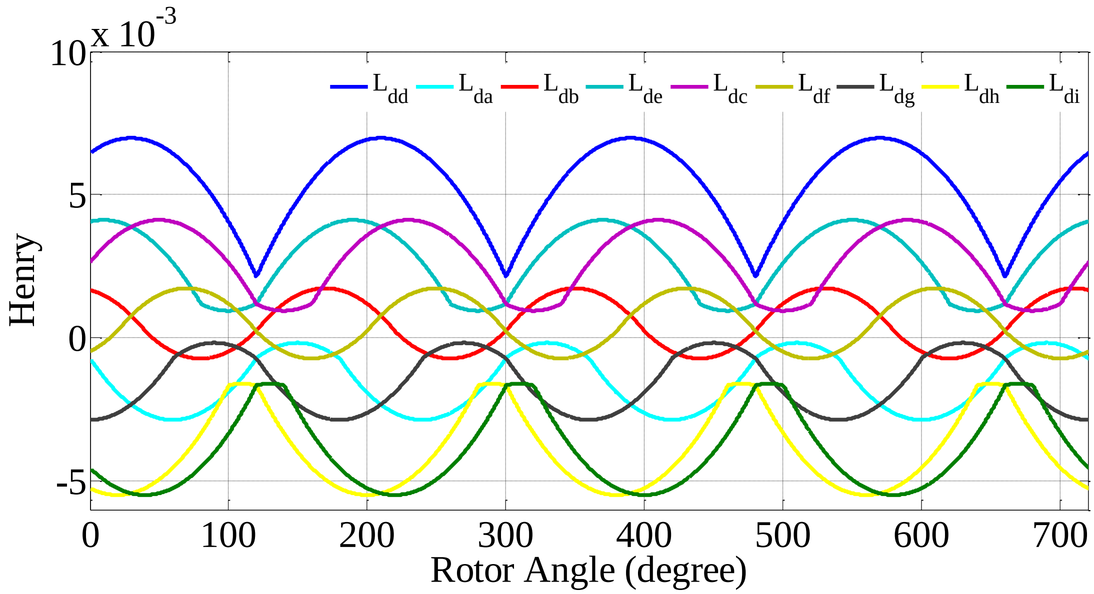


Figure 3.20: The self and mutual inductances corresponding to phase ‘d’.

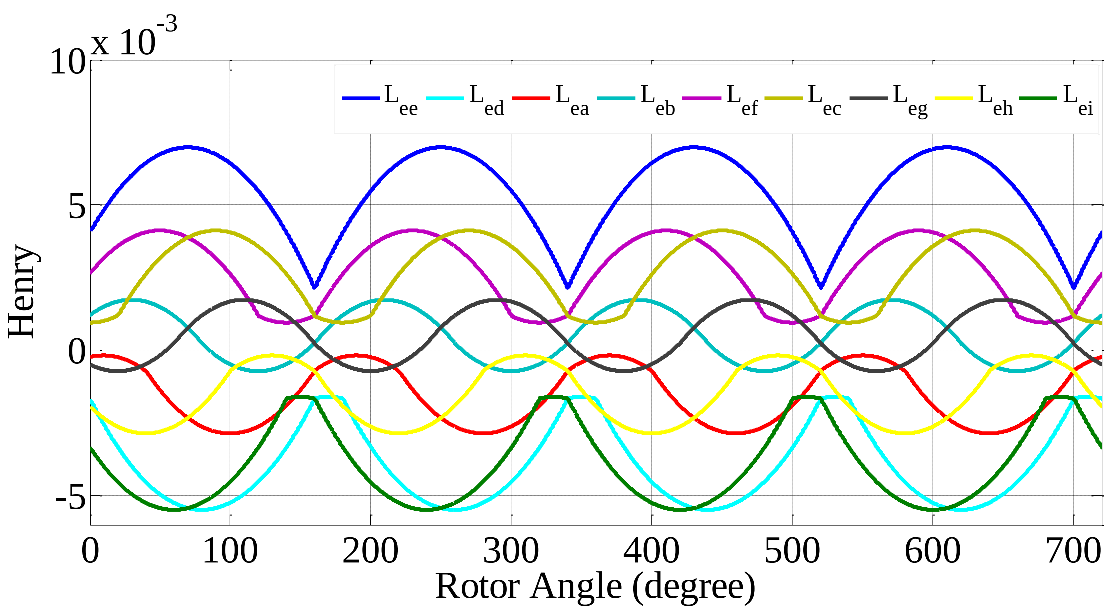


Figure 3.21: The self and mutual inductances corresponding to phase ‘e’.

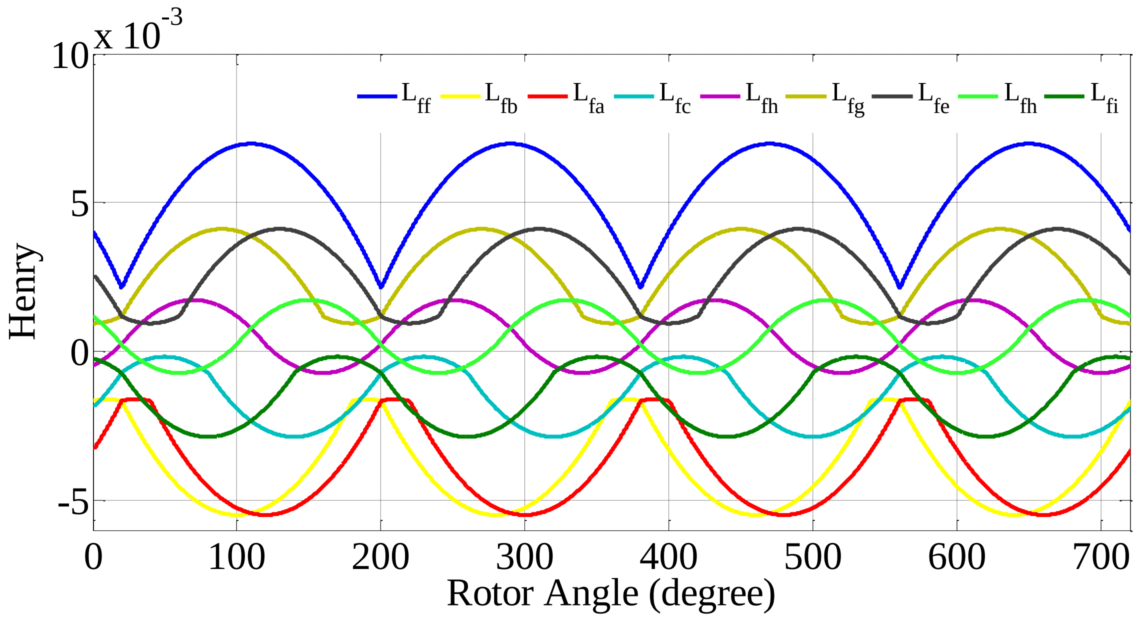


Figure 3.22: The self and mutual inductances corresponding to phase ‘f’.

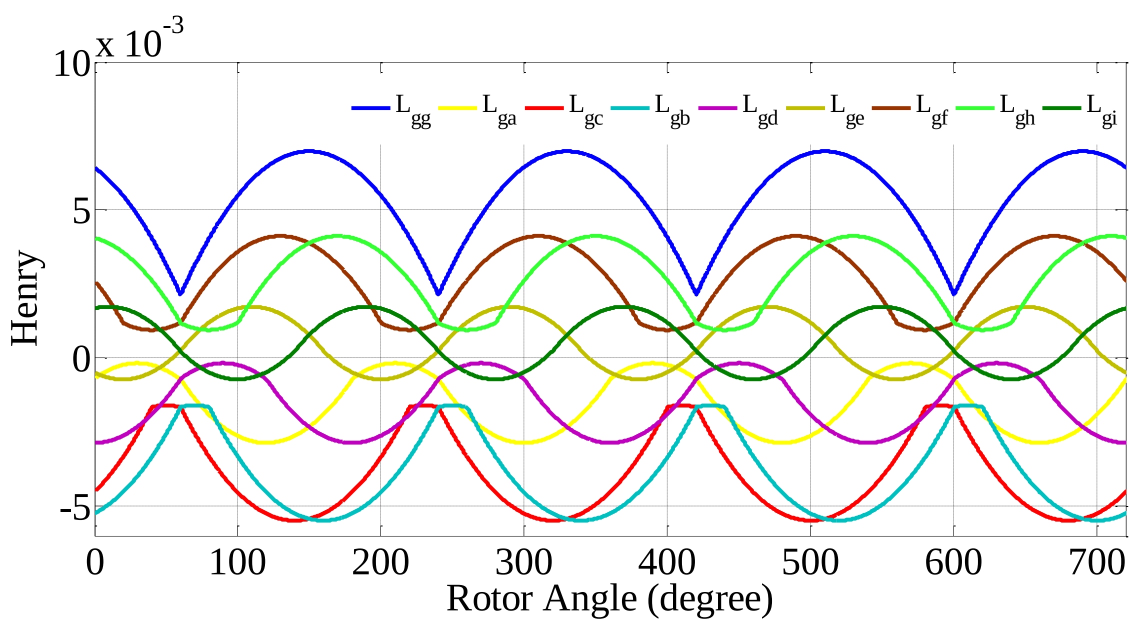


Figure 3.23: The self and mutual inductances corresponding to phase ‘g’.

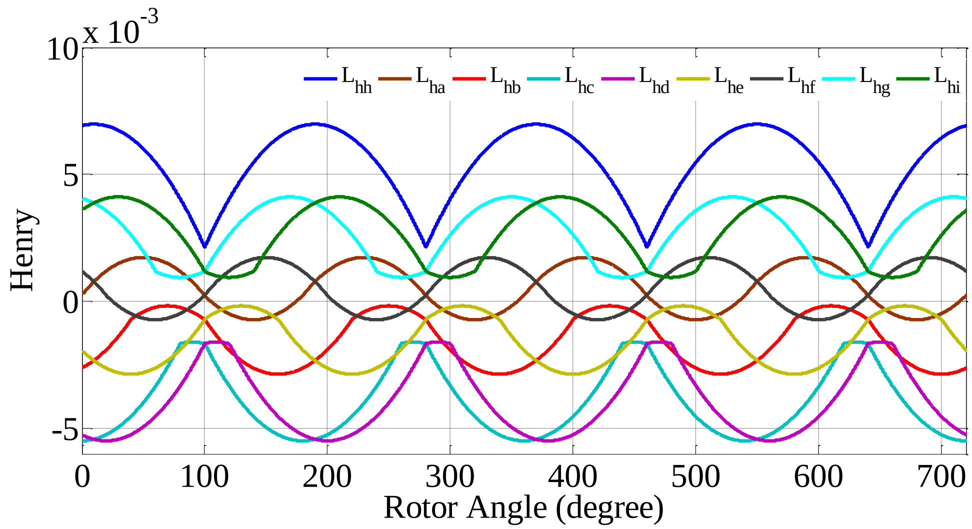


Figure 3.24: The self and mutual inductances corresponding to phase ‘h’.

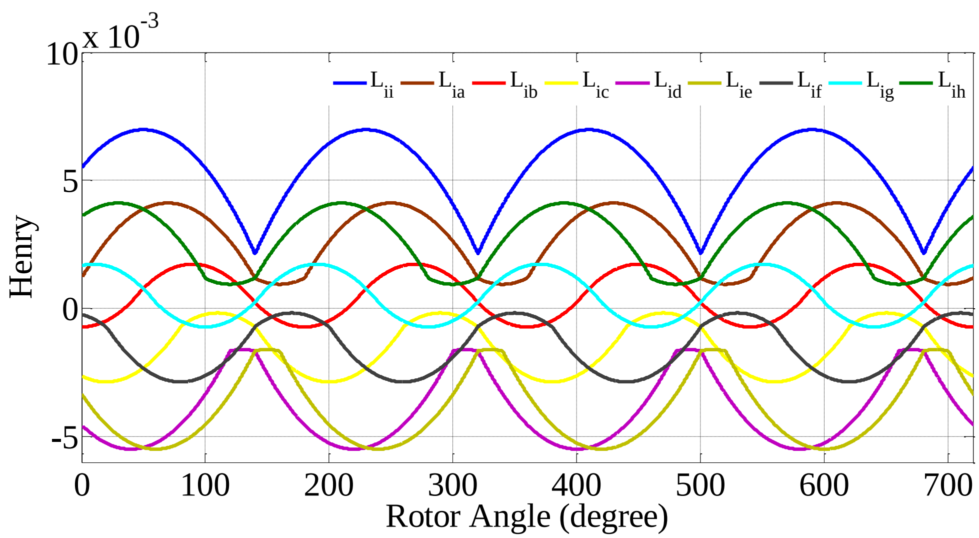


Figure 3.25: The self and mutual inductances corresponding to phase ‘i’.

Now the inductances can be arranged in the matrix of equation (3.35) and they can be transformed to the rotor reference frame to obtain the *q* and *d* inductances of the machine for different harmonics. The inductances are functions of the angle ‘ θ_r’. Stator self-inductances are maximum when the rotor q-axis is aligned with the phase winding, and mutual inductances are maximum when the rotor q-axis is in the midway between two phases.

$$L_{ss} = \begin{bmatrix} L_{ls}+L_{aa} & L_{ab} & L_{ac} & L_{ad} & L_{ae} & L_{af} & L_{ag} & L_{ah} & L_{ai} \\ L_{ba} & L_{ls}+L_{bb} & L_{bc} & L_{bd} & L_{be} & L_{bf} & L_{bg} & L_{bh} & L_{bi} \\ L_{ca} & L_{cb} & L_{ls}+L_{cc} & L_{cd} & L_{ce} & L_{cf} & L_{cg} & L_{ch} & L_{ci} \\ L_{da} & L_{db} & L_{dc} & L_{ls}+L_{dd} & L_{de} & L_{df} & L_{dg} & L_{dh} & L_{di} \\ L_{ea} & L_{eb} & L_{ec} & L_{ed} & L_{ls}+L_{ee} & L_{ef} & L_{eg} & L_{eh} & L_{ei} \\ L_{fa} & L_{fb} & L_{fc} & L_{fd} & L_{fe} & L_{ls}+L_{ff} & L_{fg} & L_{fh} & L_{fi} \\ L_{ga} & L_{gb} & L_{gc} & L_{gd} & L_{ge} & L_{gf} & L_{ls}+L_{gg} & L_{gh} & L_{gi} \\ L_{ha} & L_{hb} & L_{hc} & L_{hd} & L_{he} & L_{hf} & L_{hg} & L_{ls}+L_{hh} & L_{hi} \\ L_{ia} & L_{ib} & L_{ic} & L_{id} & L_{ie} & L_{if} & L_{ig} & L_{ih} & L_{ls}+L_{ii} \end{bmatrix} \quad (3.35)$$

The generated inductances in the rotor reference frame are shown in the Figures 3.26 to 3.29.

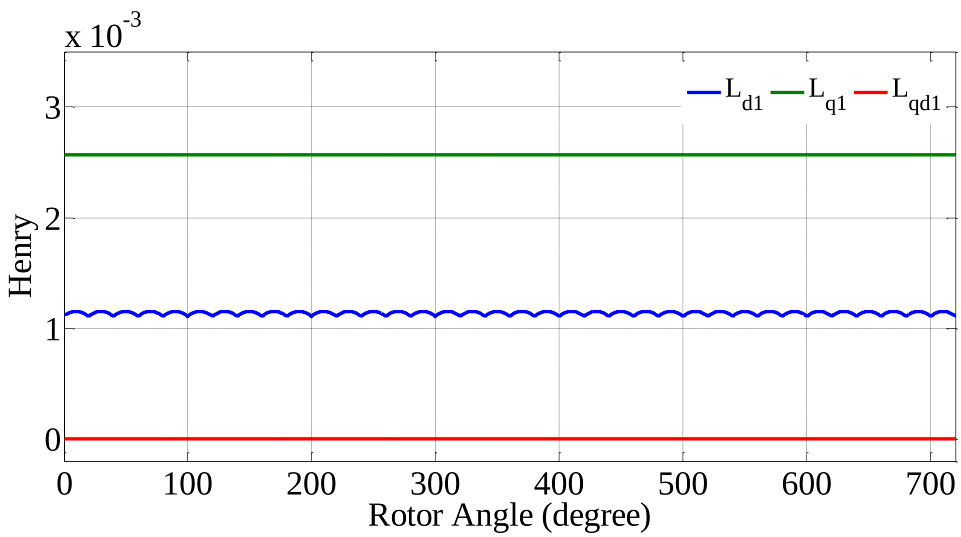


Figure 3.26: The main component of the self and mutual inductances of the machine in the rotor reference frame.

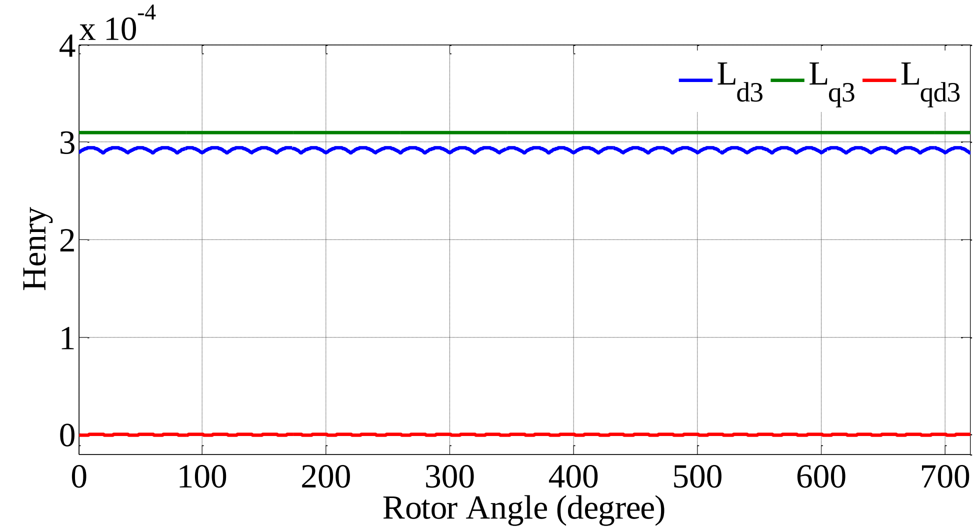


Figure 3.27: The third harmonic of the self and mutual inductances of the machine in the rotor reference frame.

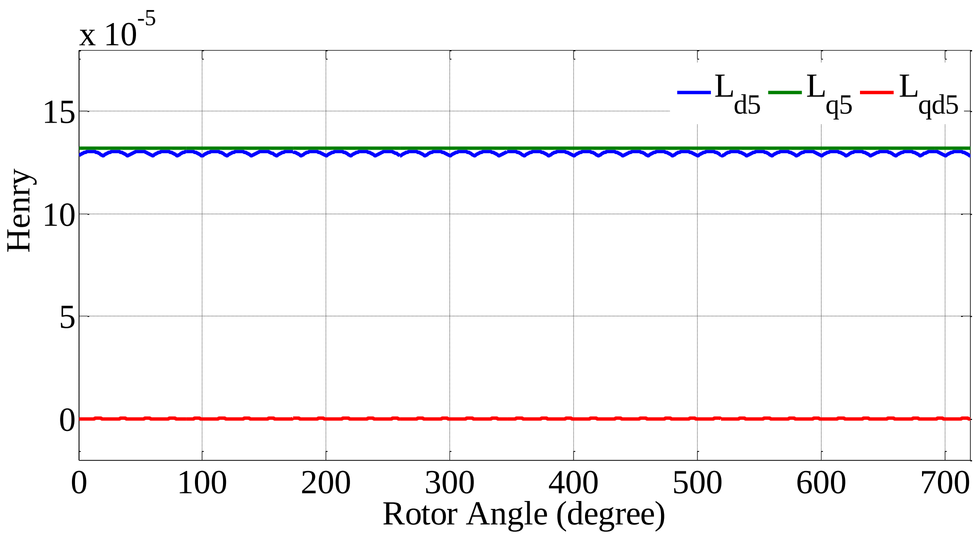


Figure 3.28: The fifth harmonic of the self and mutual inductances of the machine in the rotor reference frame.

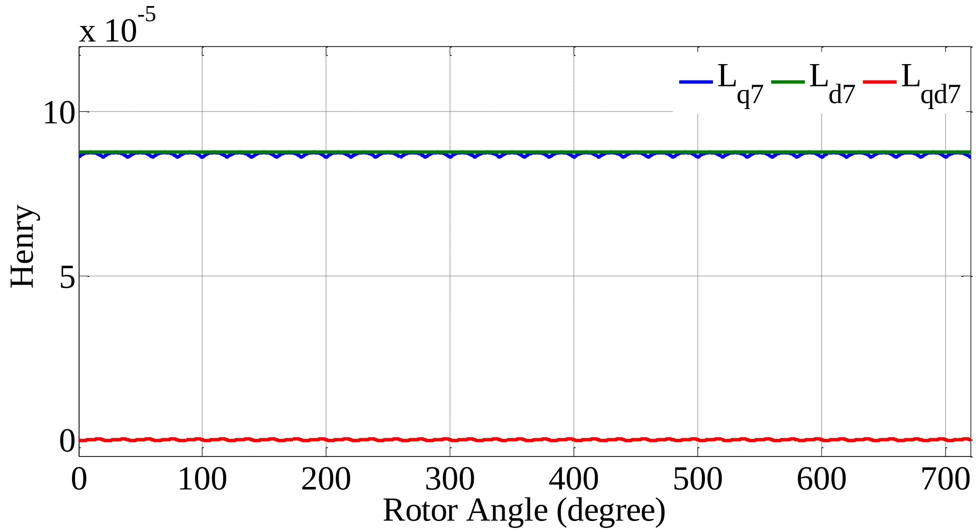


Figure 3.29: The seventh harmonic of the self and mutual inductances of machine in the rotor reference frame.

The flux linkage due to the rotor permanent magnets seen from the stator phases could be expressed as equation (3.36) [102]. Figure 3.30 shows the rotor flux linkage seen from the phase 'a'. The flux linkage seen from the rest of the phases have the same shape with 40 degrees' phase shift in spatial angle from the adjacent phases [83].

$$B_r(\theta_r) = \begin{cases} B_{\max} & -\dfrac{90^o - \alpha_p}{2} < |\theta_r| < \dfrac{90 - \alpha_p}{2}\ ,\ 135^o - \alpha_p < |\theta_r| < 180^o \\ B_{\max} - \dfrac{2B_{\max}}{\alpha_p}\theta_r & \dfrac{90^o - \alpha_p}{2} < |\theta_r| < 90^o \\ -B_{\max} & 90^o < |\theta_r| < 135^o - \dfrac{3\alpha_p}{2} \\ -B_{\max} + \dfrac{2B_{\max}}{\alpha_p}\theta_r & 135^o - \dfrac{3\alpha_p}{2} < |\theta_r| < 135^o - \alpha_p \end{cases} \tag{3.36}$$

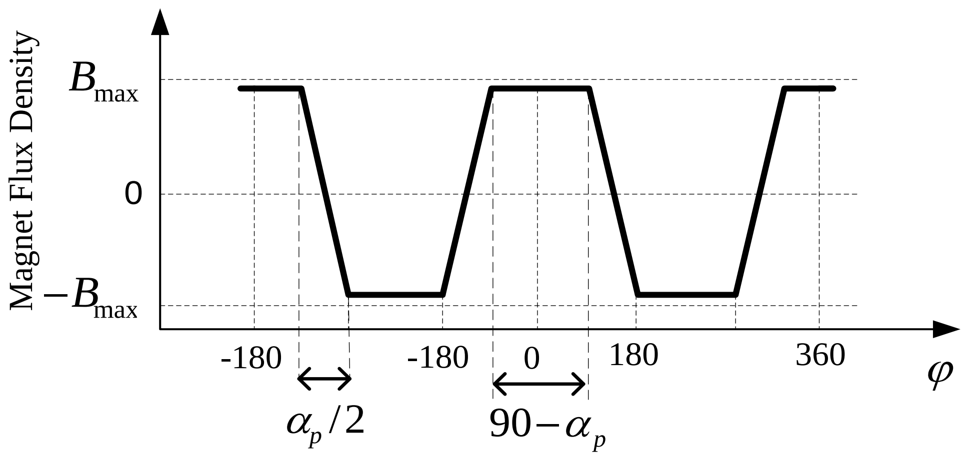


Figure 3.30: Plot of flux density with spatial angle.

Now according to equation (3.19) the flux linkage due to the permanent magnets on each phase of the stator can be transformed to the rotor reference frame to obtain the *q* and *d* axis flux linkages for different harmonics. Figures 3.31 to 3.38 show the flux linkage components.

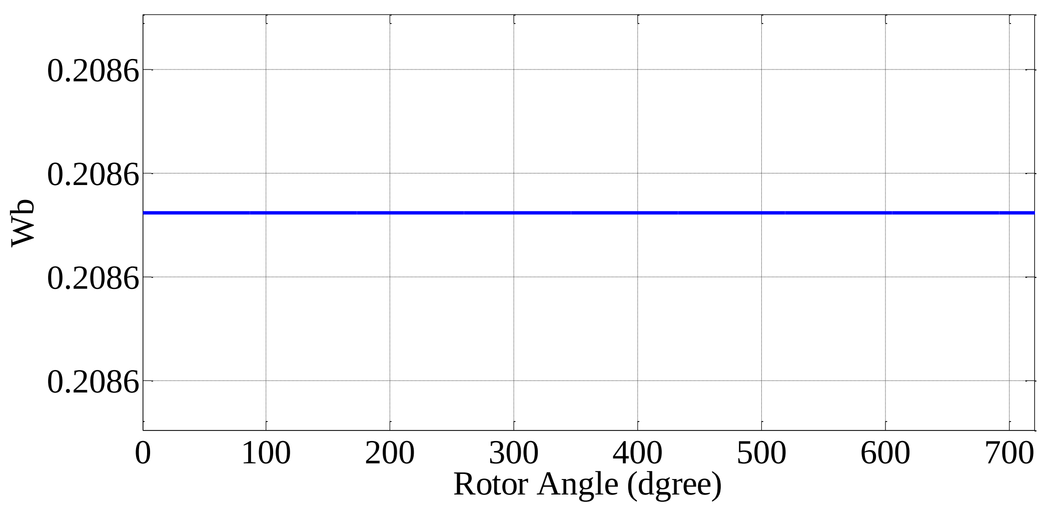


Figure 3.31: The first harmonic of the permanent magnet flux linkage in the d axis of the rotor reference frame.

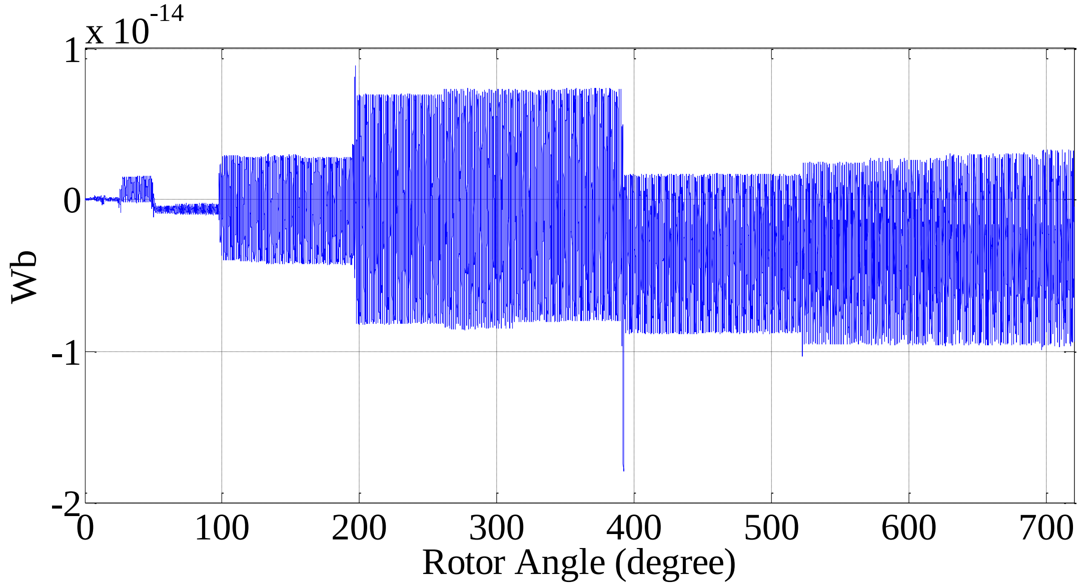


Figure 3.32: The first harmonic of the permanent magnet flux linkage in the q axis of the rotor reference frame.

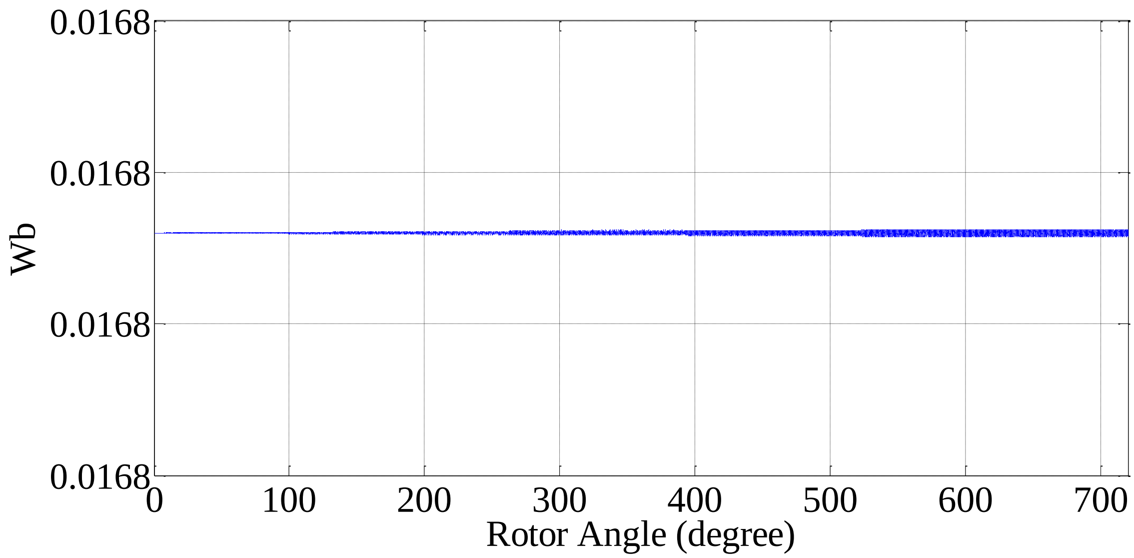


Figure 3.33: The third harmonic of the permanent magnet flux linkage in the d axis of the rotor reference frame.

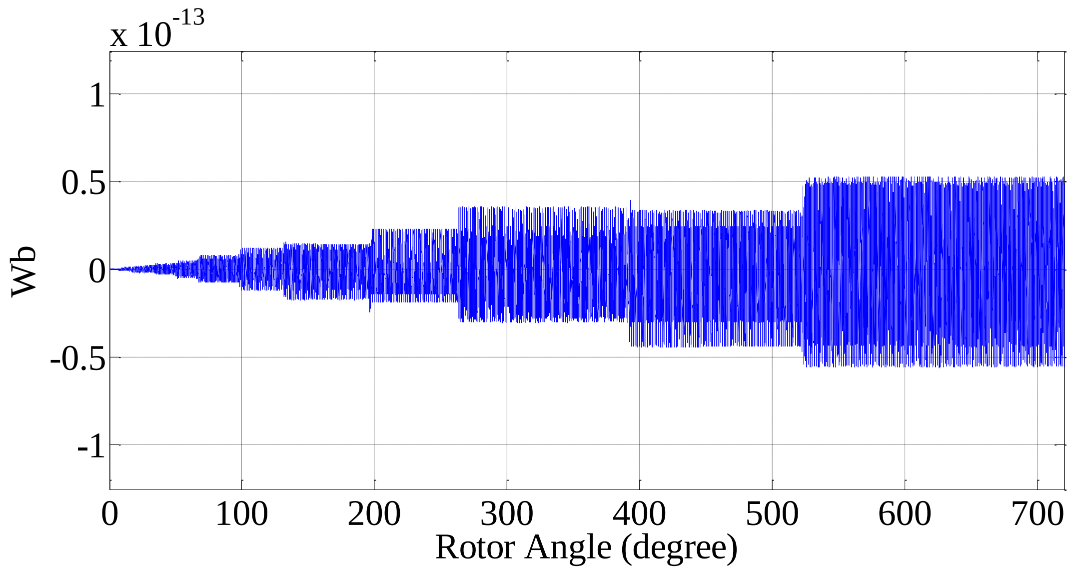


Figure 3.34: The third harmonic of the permanent magnet flux linkage in the q axis of the rotor reference frame.

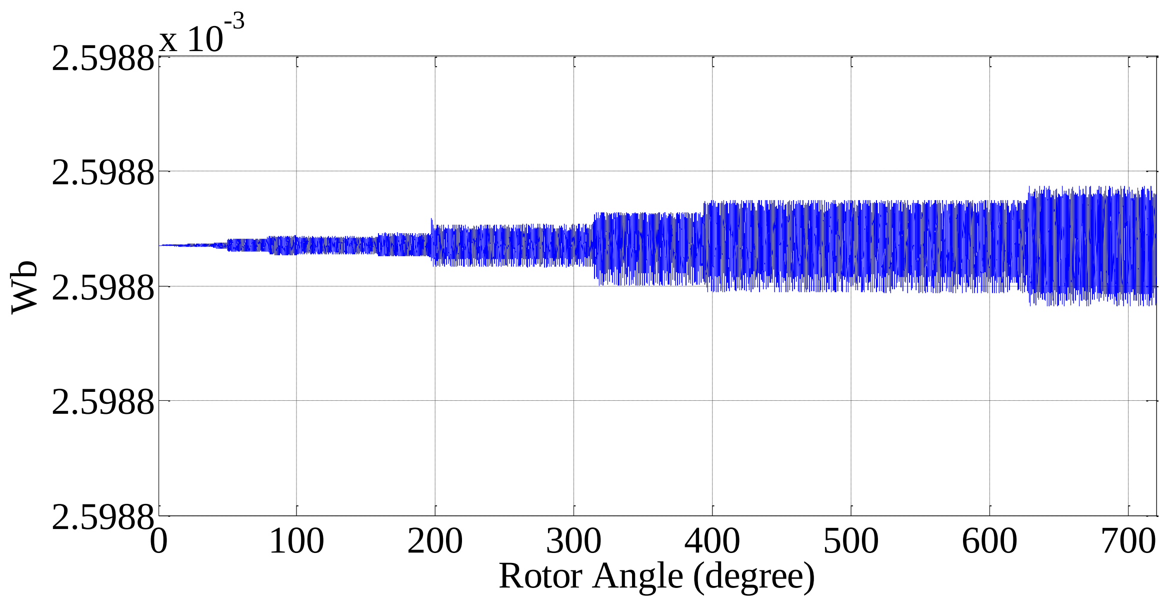


Figure 3.35: The fifth harmonic of the permanent magnet flux linkage in the d axis of the rotor reference frame.

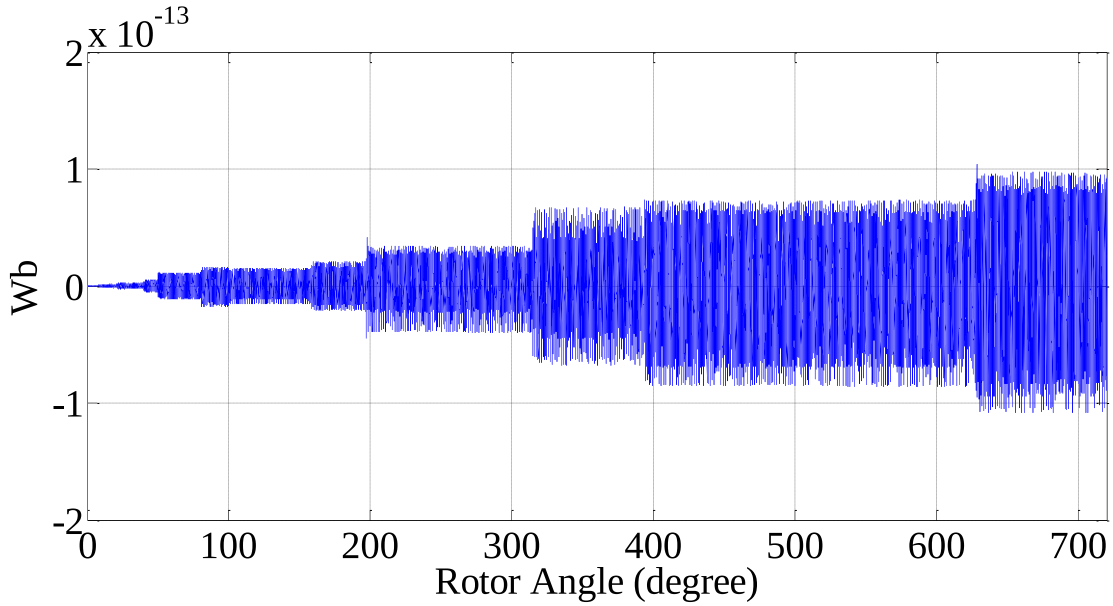


Figure 3.36: The fifth harmonic of the permanent magnet flux linkage in the q axis of the rotor reference frame.

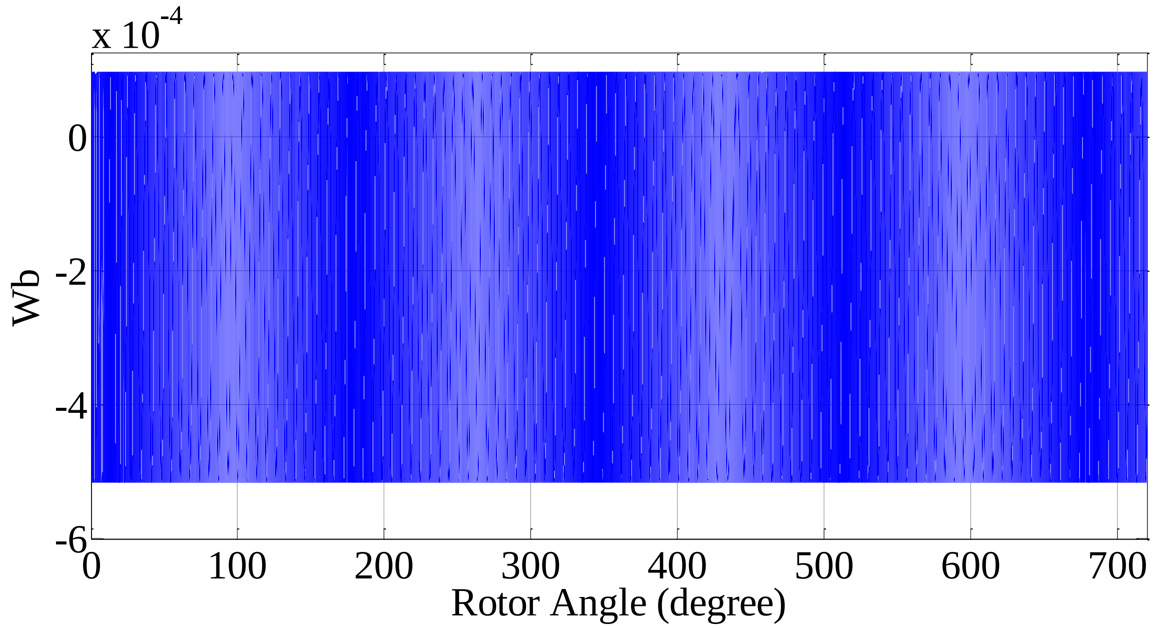


Figure 3.37: The seventh harmonic of the permanent magnet flux linkage in the d axis of the rotor reference frame.

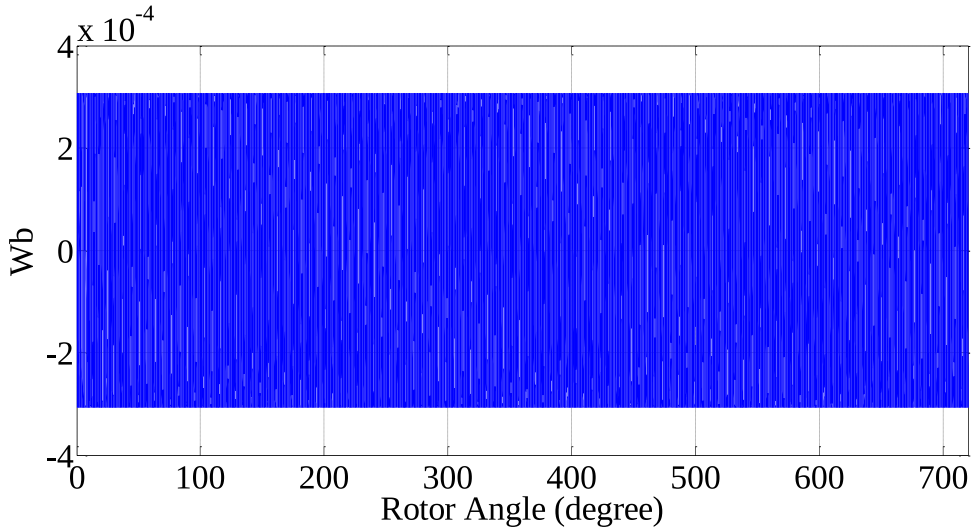


Figure 3.38: The seventh harmonic of the permanent magnet flux linkage in the q axis of the rotor reference frame.

3.3 Simulation of the Coupled Model of the Nine-Phase IPM

The generated components can be used to model the machine. Using all generated equations and wave forms for the machine and substituting the machine parameters presented in the Table 3.1 [163], the nine-phase IPM machine can be simulated using MATLAB/ Simulink.

A 60 (Hz) 110 (Volts) nine-phase voltage (as shown in Figure 3.39) is applied to the model and while the initial rotor speed is 377 rad/sec. When the machine passes the transients and goes to the steady state, a mechanical load torque equal to 3 N.m is applied to the machine.

The following figures show the simulation results. Figure 3.40 shows the rotor speed it can be seen that after initial transients the rotor speed goes to the synchronous speed which is equal to the source frequency. Also when the load torque is applied to the machine, due to the change in the torque angle of the machine the speed shows transients before going back to the steady state.

Table 3.1 Design Parameters of the Nine-phase IPM.

Stator Data:	
Outer Diameter of Stator	154.432 (mm)
Inner Diameter of Stator	93.4212 (mm)
Number of Stator slots	36 (mm)
Stator Slot Dimensions	
H_{s0}	0.508 (mm)
H_{s1}	0.762 (mm)
H_{s2}	11.43 (mm)
B_{s0}	1.9304 (mm)
B_{s1}	4.191 (mm)
B_{s2}	6.39572 (mm)
Top Tooth Width	4.18319 (mm)
Bottom Tooth Width	3.97339 (mm)
Length of Stator Core	95.25 (mm)
Stacking Factor of Stator Core	0.93
Magnet Duct Dimensions	
D1 (Length)	69.073 (mm)
O1(Width)	23.5331 (mm)
Rib	3.302 (mm)
Magnet type	Samarium Cobalt
Magnet Thickness	6.35 (mm)
Total Magnetic Field Strength	32.3088 (mm)
Residual Flux density	629 (kA/m)
Coercive Force	0.85 (Wb)
Maximum Energy Density	133.67 $\left(KJ.m^{3}\right)$
Relative Recoil Permeability	1.07528 $\left(H.m^{-1}\right)$
Demagnetized Flux density	0.531245 (T)
L_{q1}	$7.8\times10^{-2}(H)$
L_{d1}	$3.0\times10^{-2}(H)$
R_s	$0.01(\Omega)$

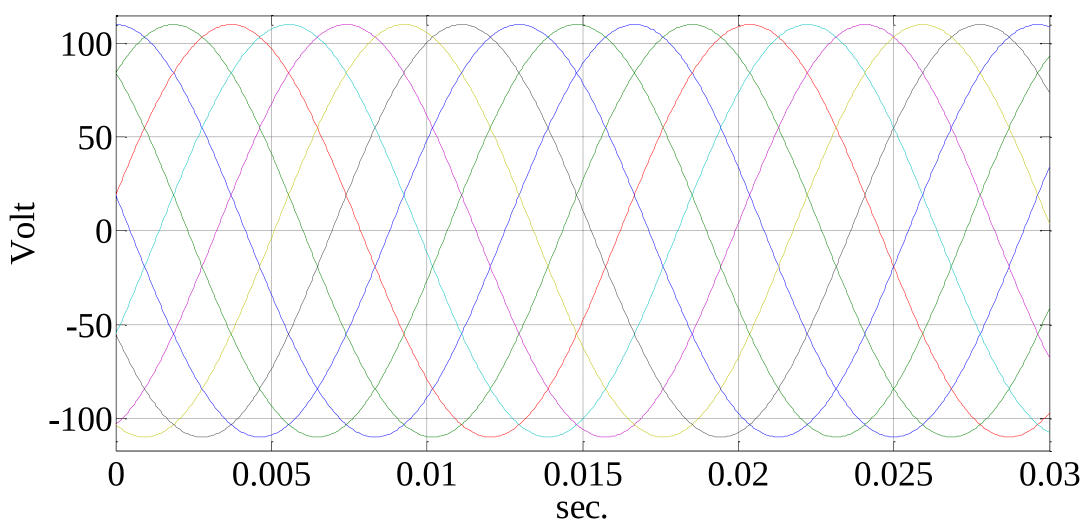


Figure 3.39: The nine- phase voltages.

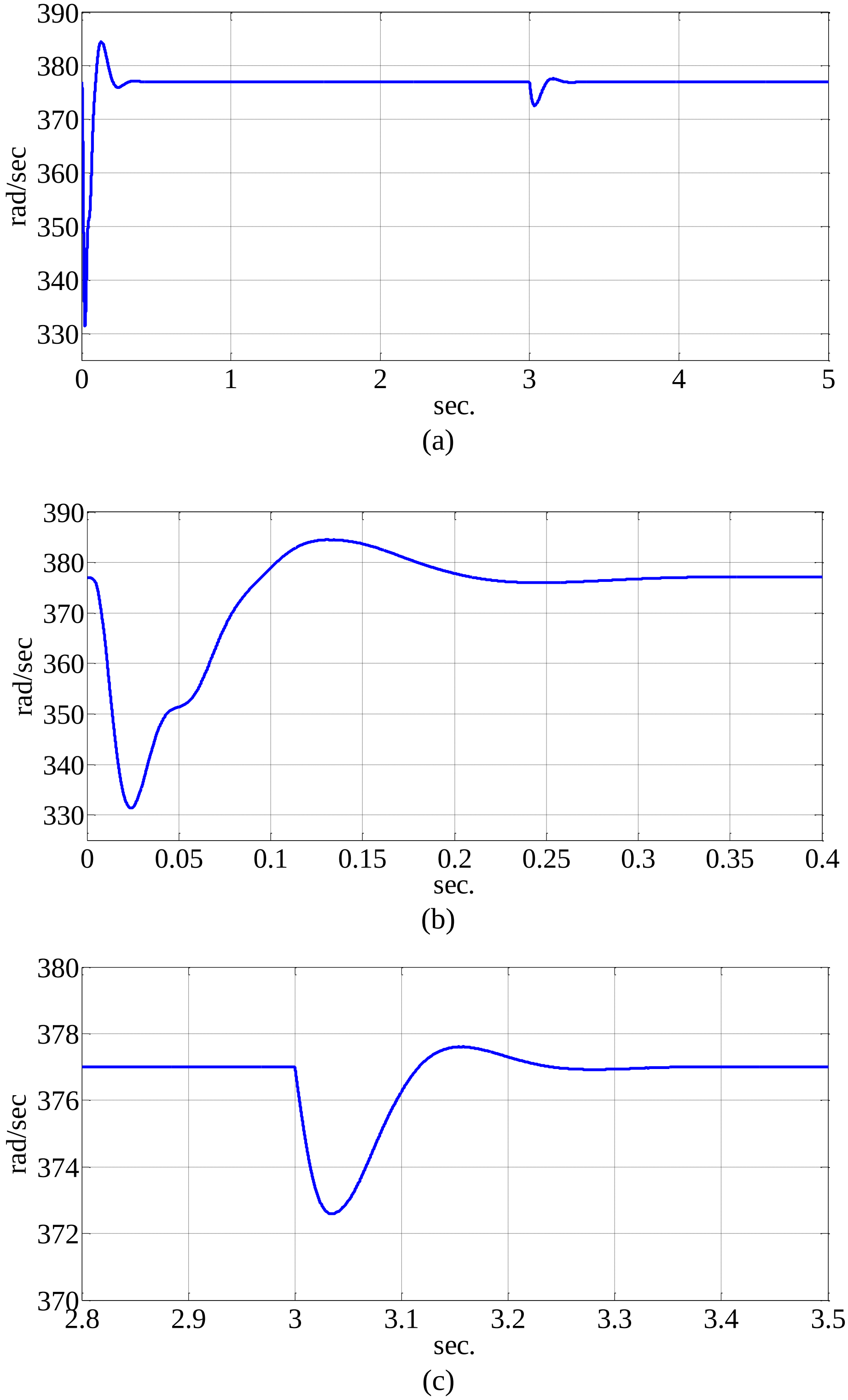
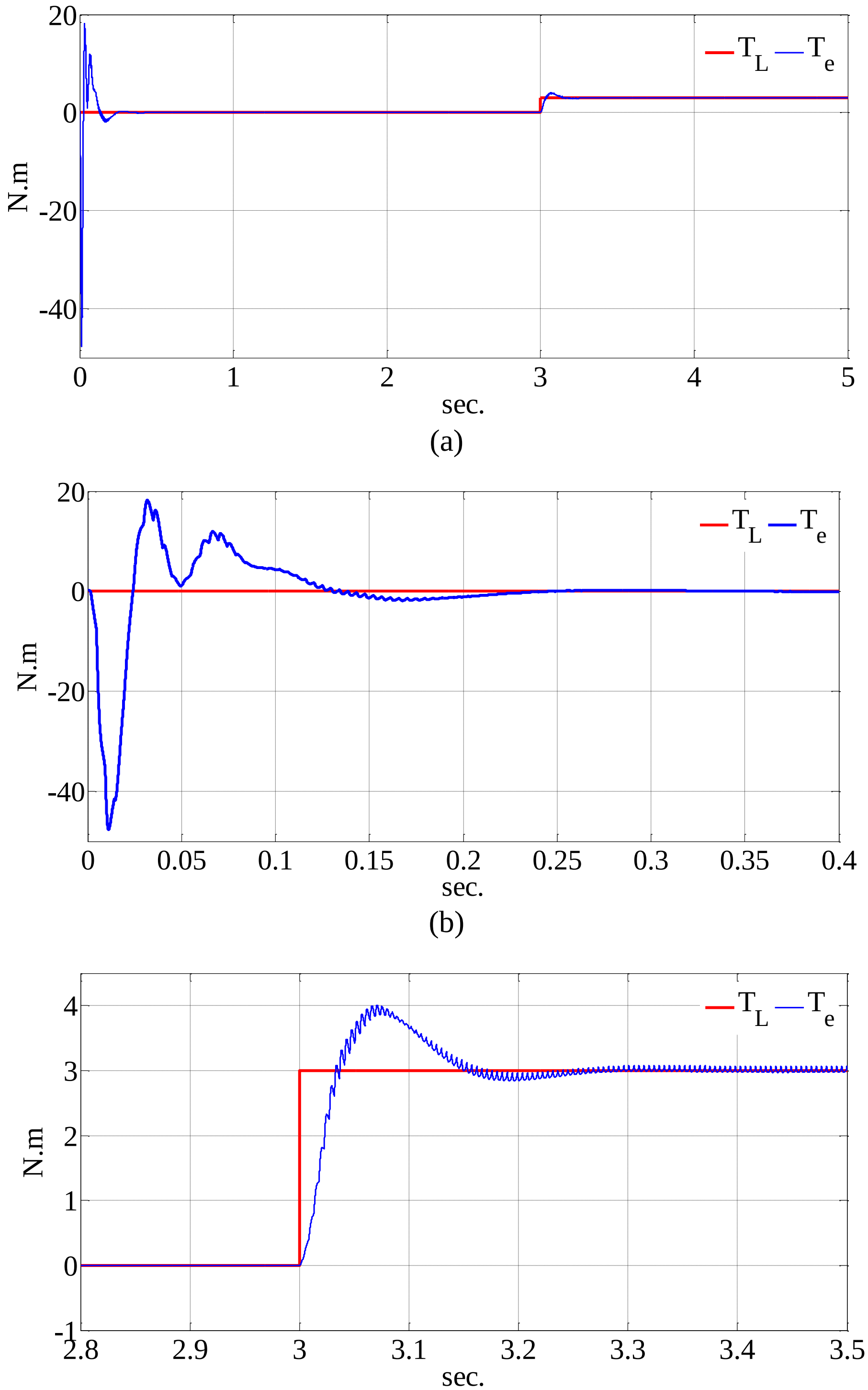


Figure 3.40: (a) The rotor speed, (b) Starting moments, (c) Load changing moments.



(c)

Figure 3.41: (a) The electromagnetic and load torque, (b) Starting moments, (c) Load change moments.

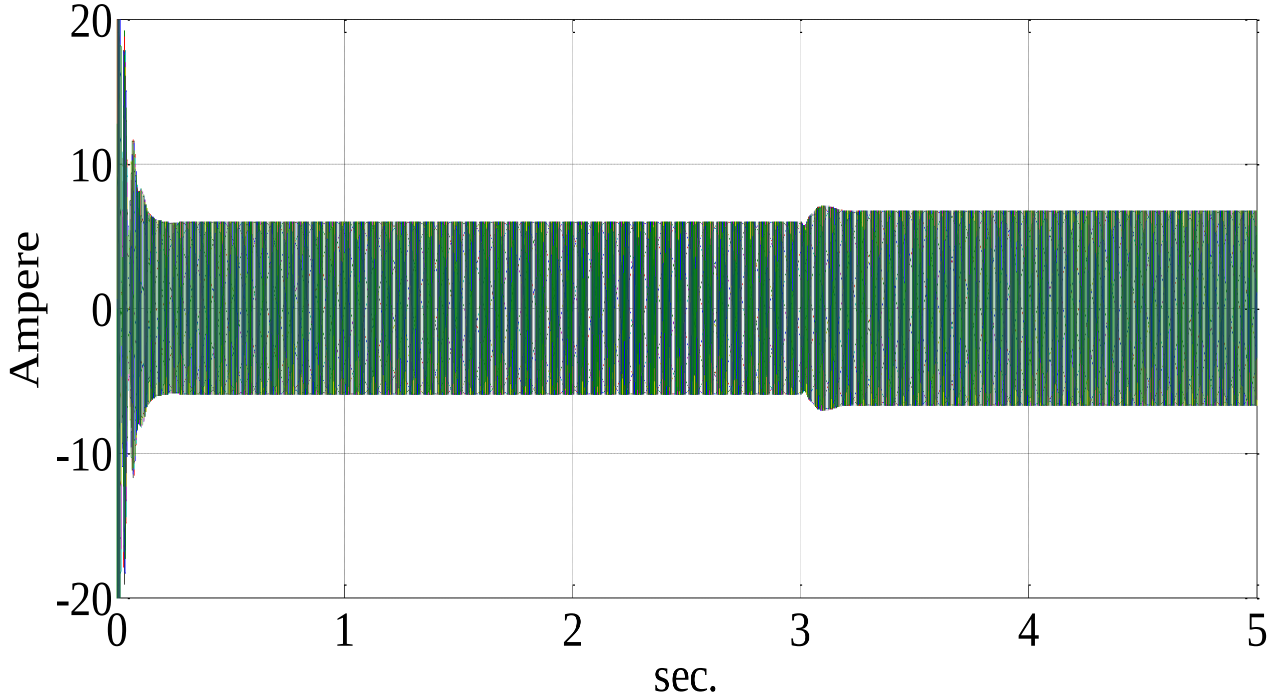
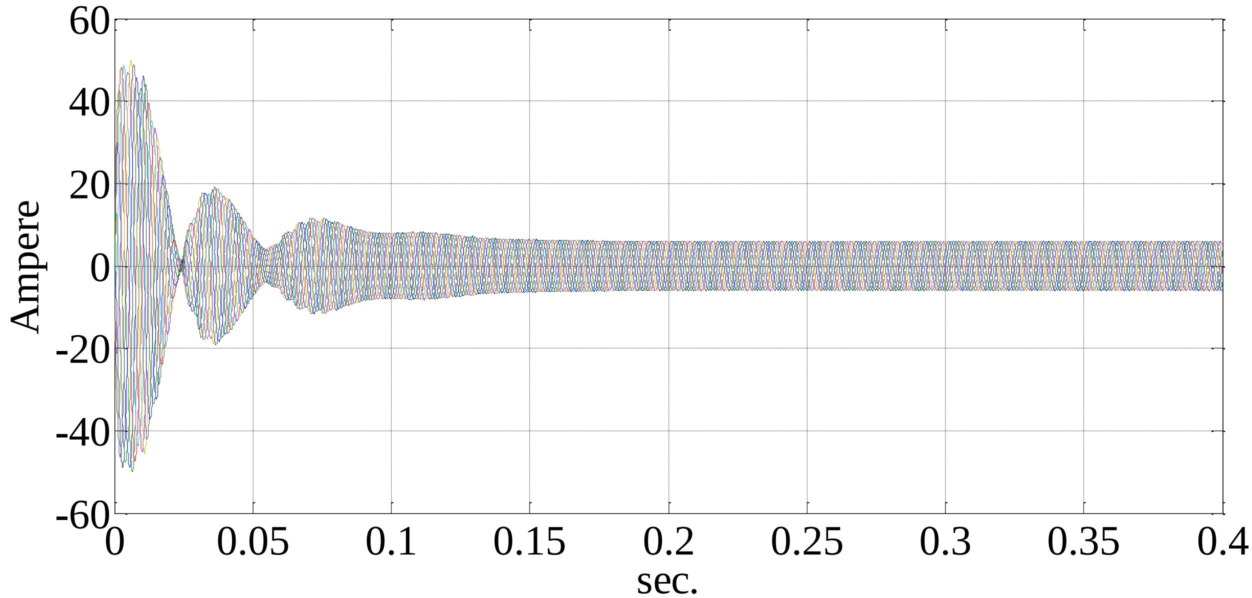
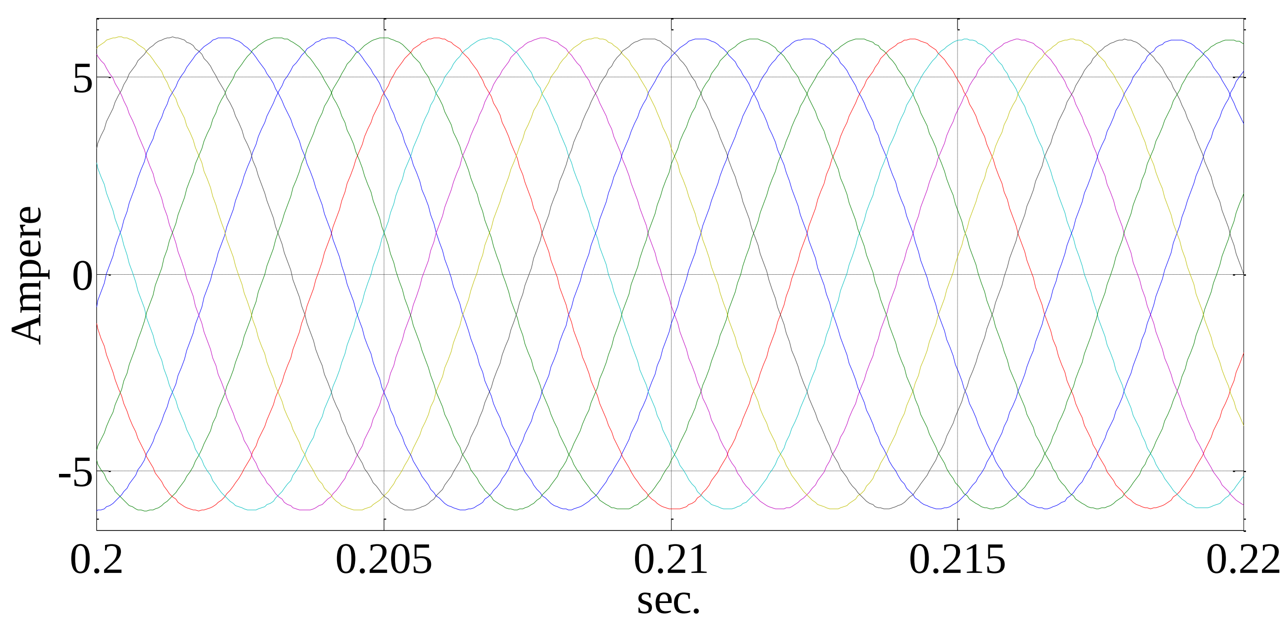


Figure 3.42: The stator currents.

Figure 3.41 show the electromagnetic and load torque together. At the initial point the machine has some transients and after a while when the transients are over the electromagnetic torque goes to zero. After applying the load, the machine starts generating electromagnetic torque to keep the synchronous speed. The stator currents are shown in the Figure 3.42. The transients can be seen at the starting and at the load change moment, when transient is passed and the machine goes to steady state the currents also settle down to their final values. To see more details, the currents are shown in the Figure 3.43 and 3.44 during their transients and steady state.

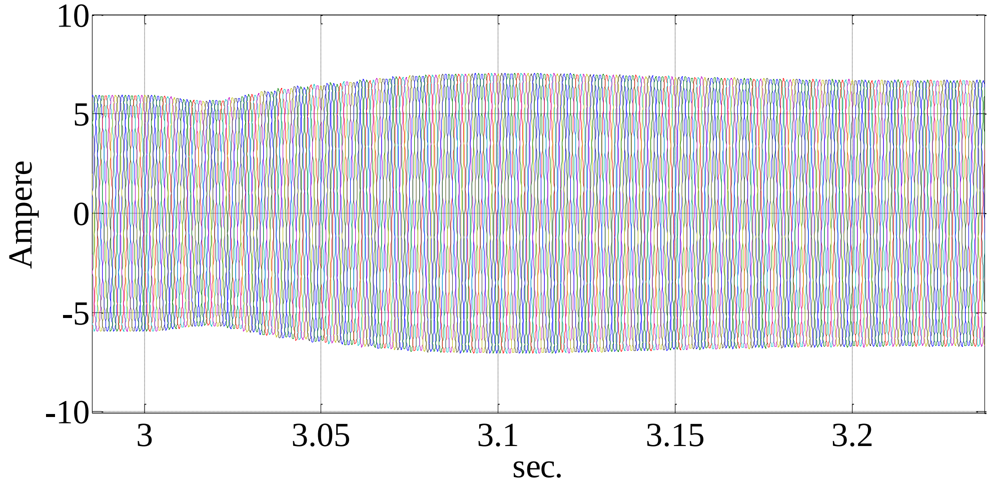


(a)

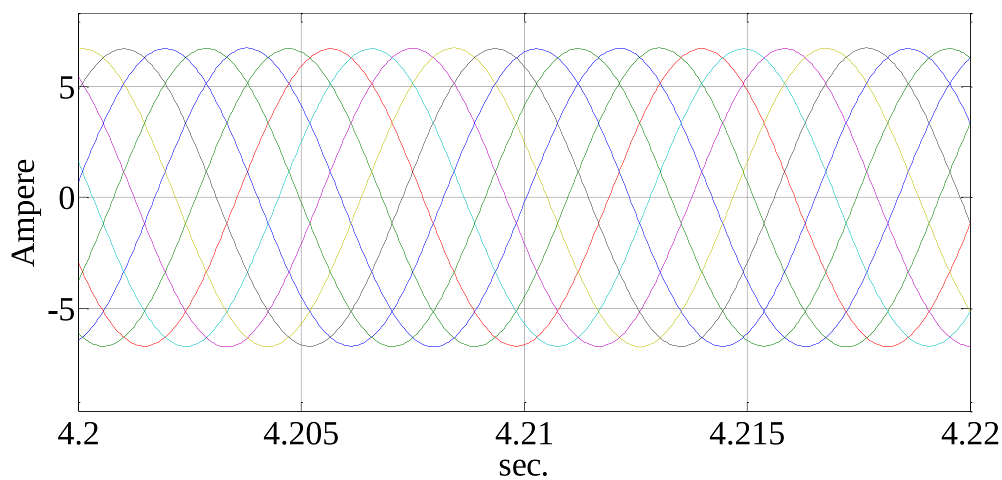


(b)

Figure 3.43: (a) The stator currents at transients before applying load torque, (b) The stator currents at steady state before applying the load torque.



(a)



(b)

Figure 3.44: (a) The stator currents at transients after applying load torque, (b) The stator currents at steady state after applying load torque.

The flux linkages of the machine in the rotor reference frame are also shown in Figures 3.45 to 3.52. Figures 3.45 and 3.46 show the main component of the *q* and *d* axis flux linkages. The transients for the starting and load changing moments are also shown. The flux linkages are sum of the flux linkages due to the permanent magnet and stator currents.

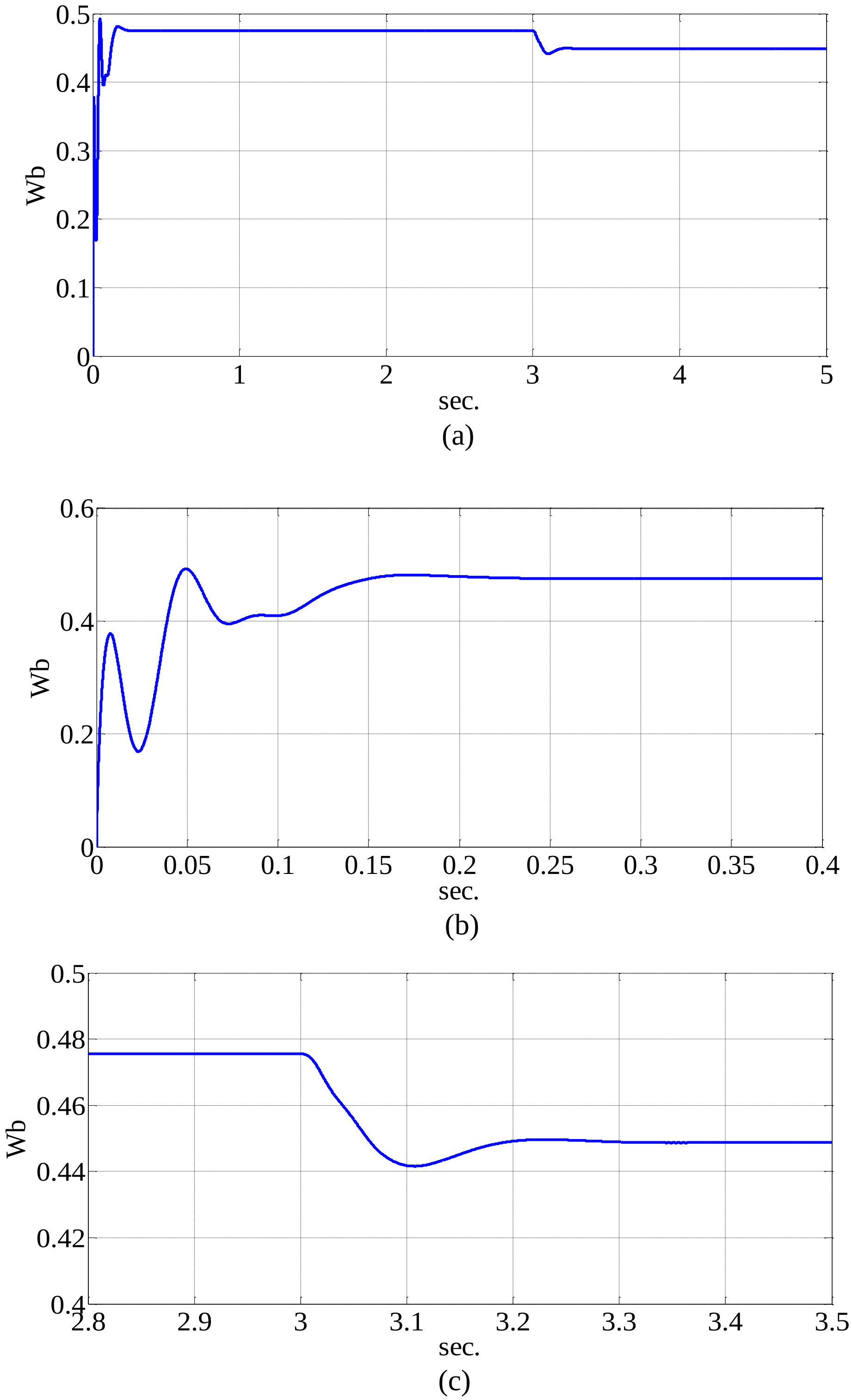
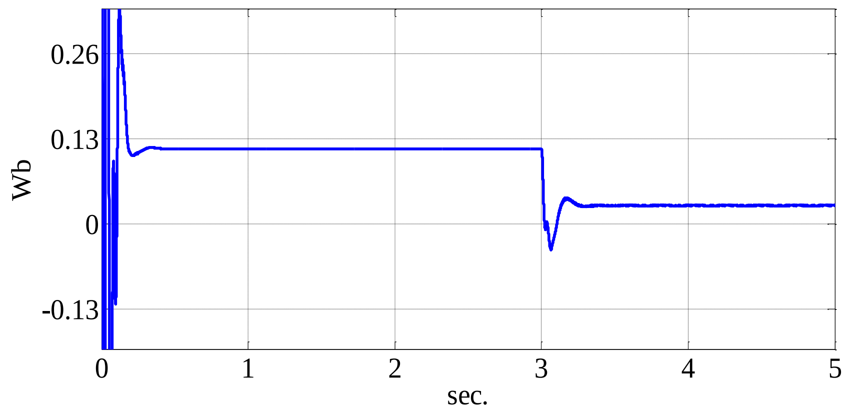
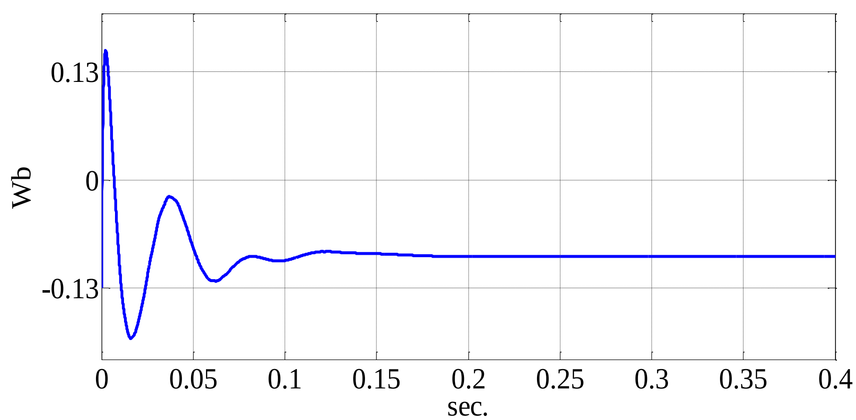


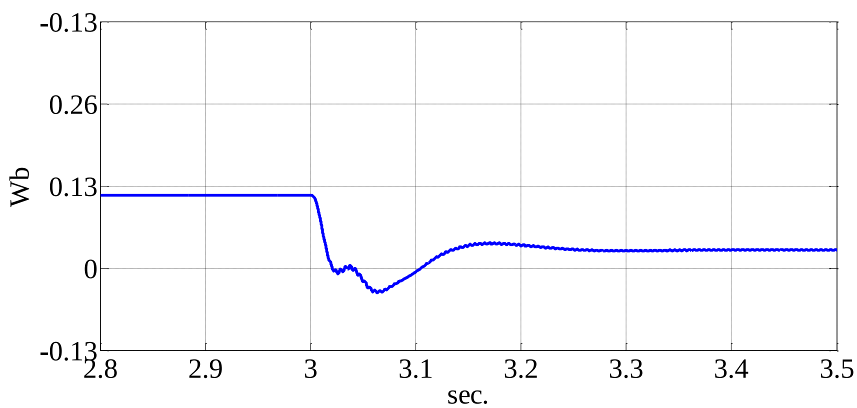
Figure 3.45: (a) The main component of the d axis flux linkage, (b) Starting moments, (c) Load changing moments.



(a)



(b)



(c)

Figure 3.46: (a) The main component of the q axis flux linkage, (b) Starting moments, (c) Load changing moments.

The third, fifth and seventh harmonics of the flux linkages are shown in the Figure 3.47 to 3.50. The magnitude of the third, fifth and seventh harmonics of the stator voltages and the currents are zero, the flux linkages of these harmonics are equal to the corresponding harmonics of the flux linkages due to the permanent magnets. It should be noted that these figures are showing the simulation results.

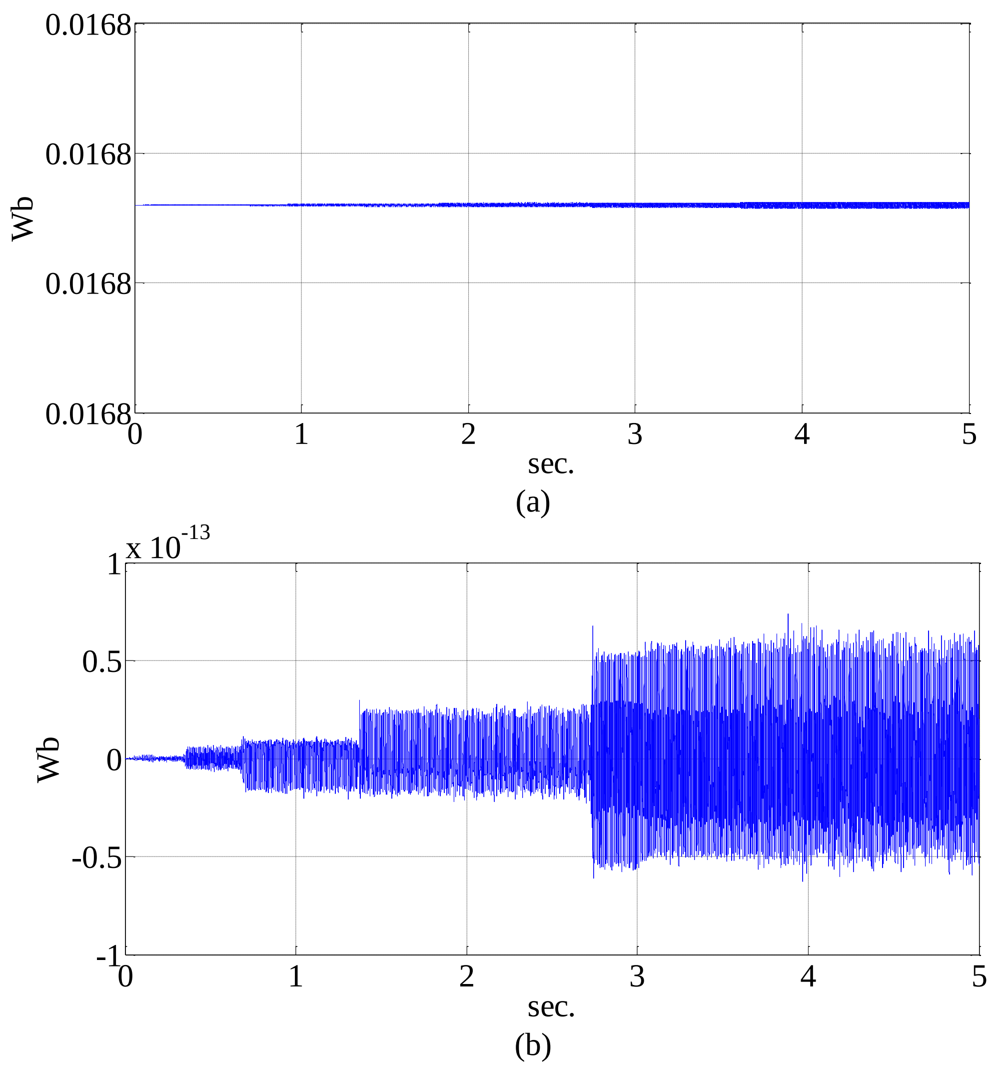


Figure 3.47: (a) The third component of the d axis flux linkage, (b) The third component of the q axis flux linkage.

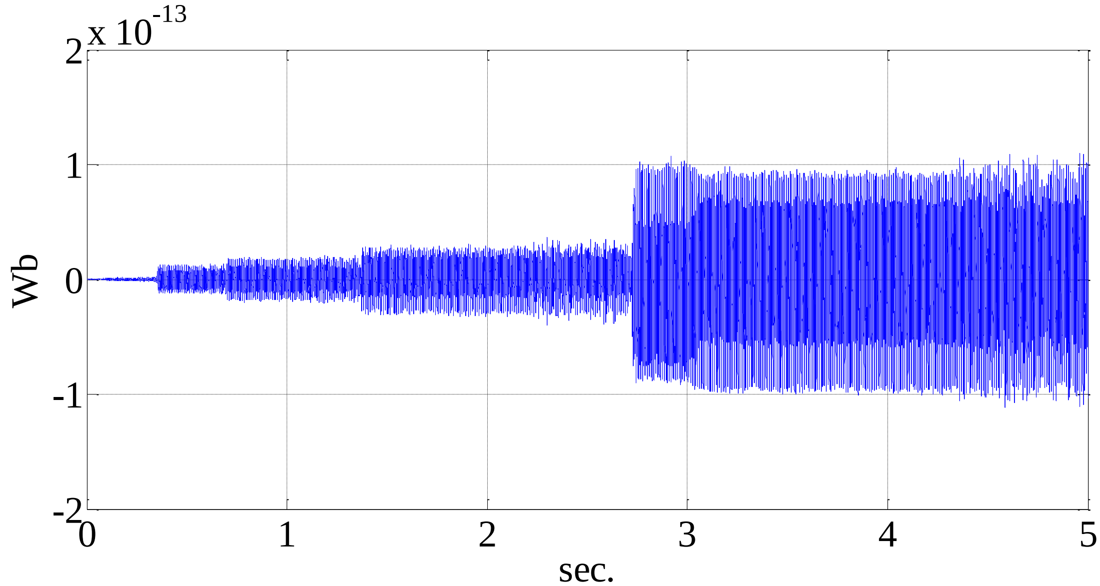


Figure 3.48: The fifth component of the d axis flux linkage.

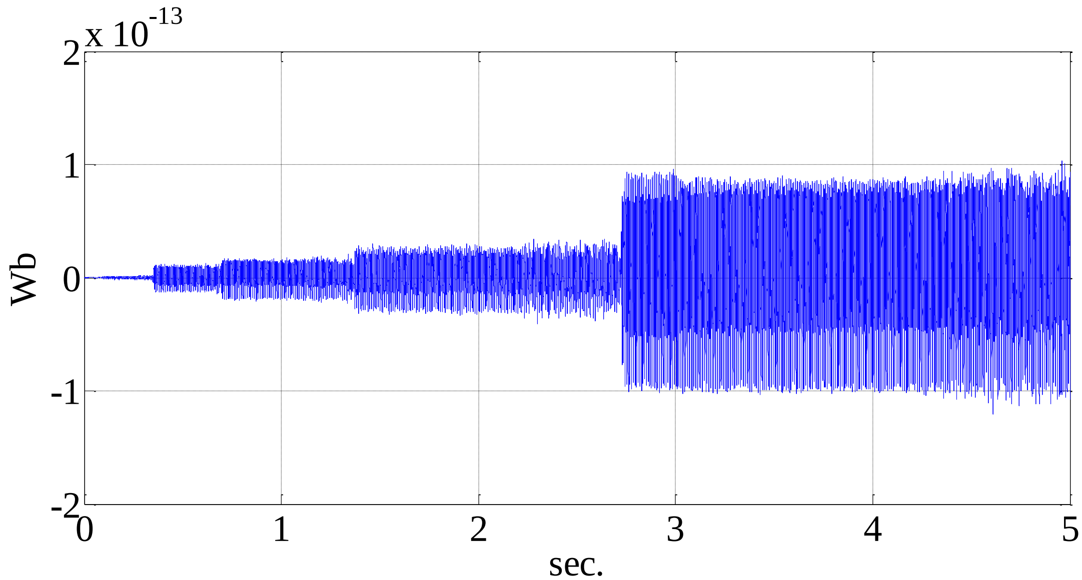


Figure 3.49: The fifth component of the q axis flux linkage.

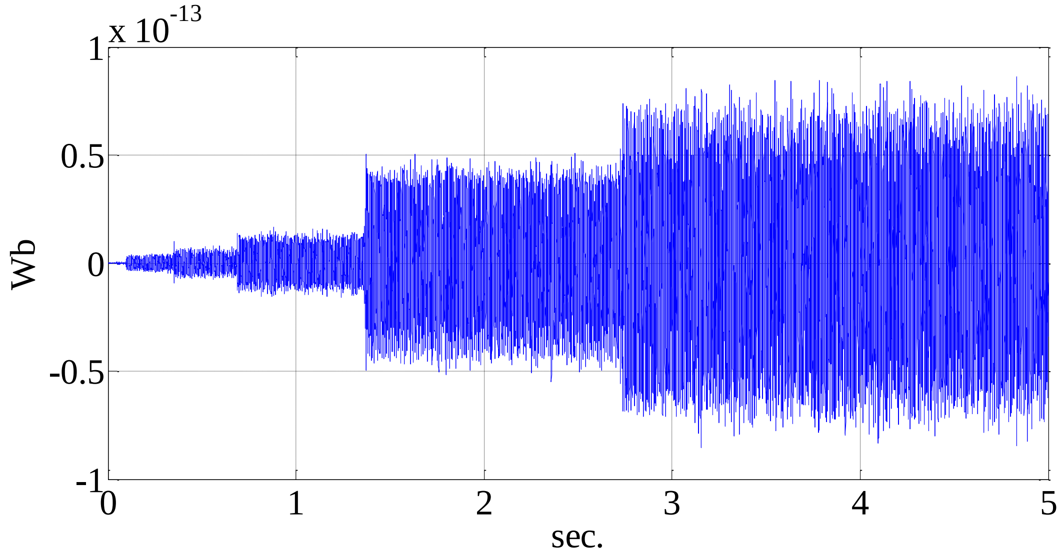


Figure 3.50: The seventh component of the d axis flux linkage.

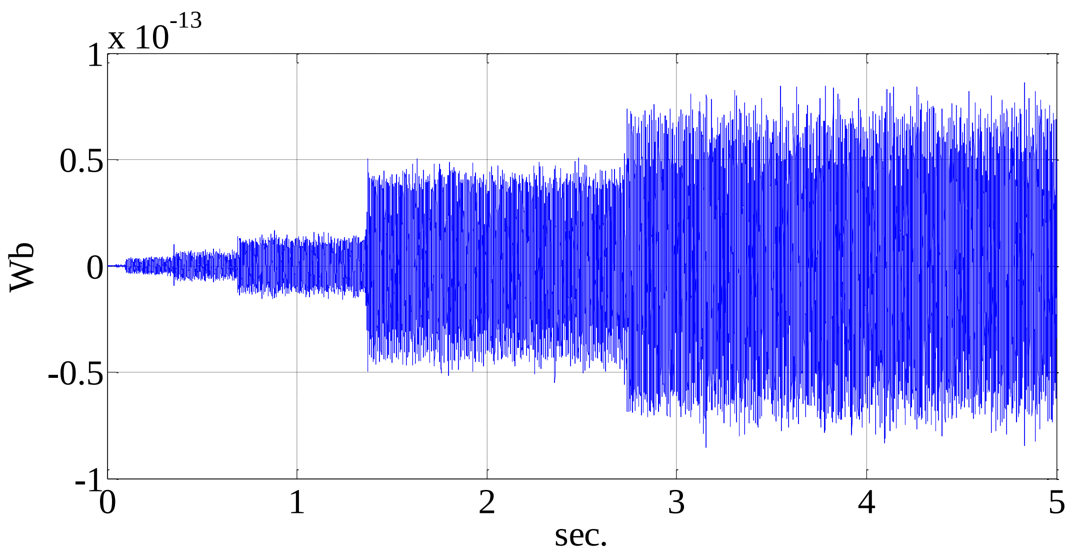


Figure 3.51: The seventh component of the q axis flux linkage.

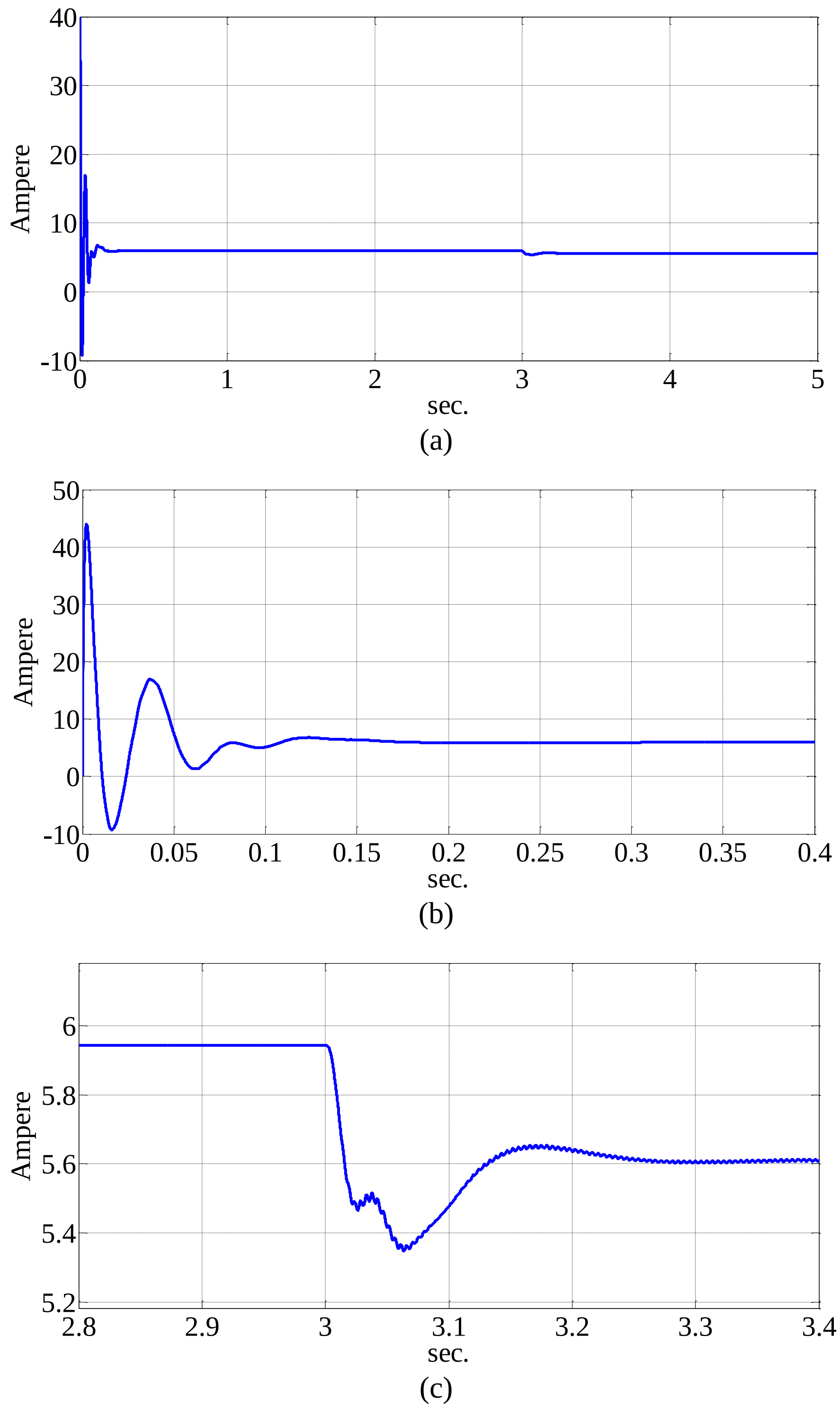
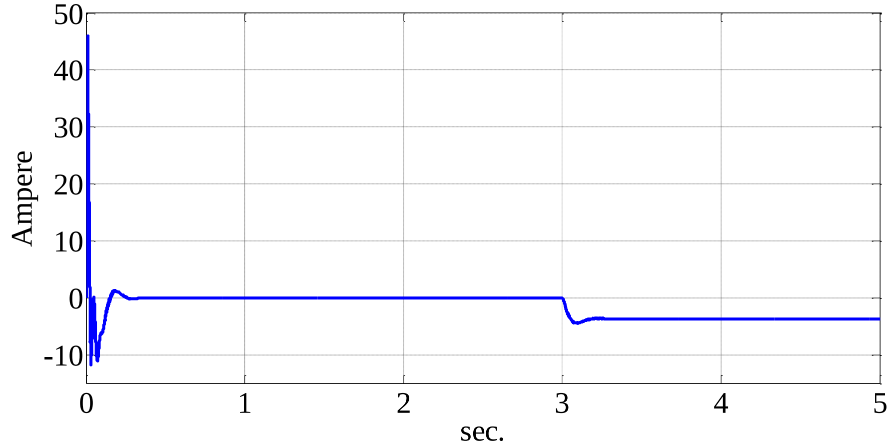
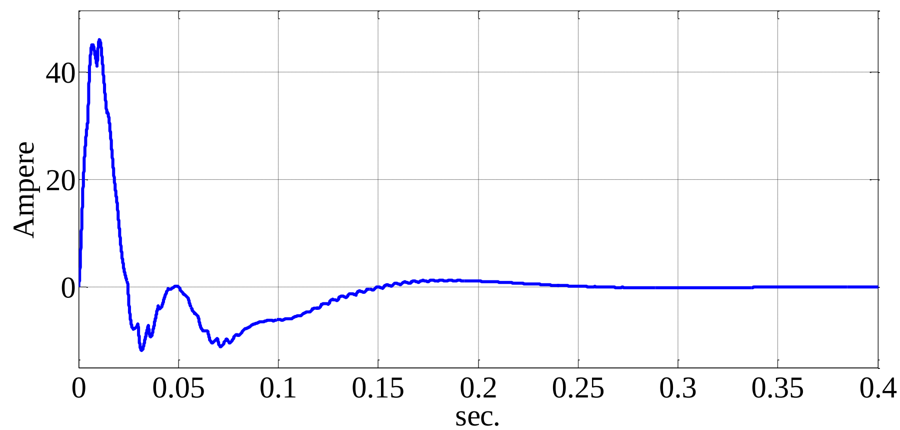


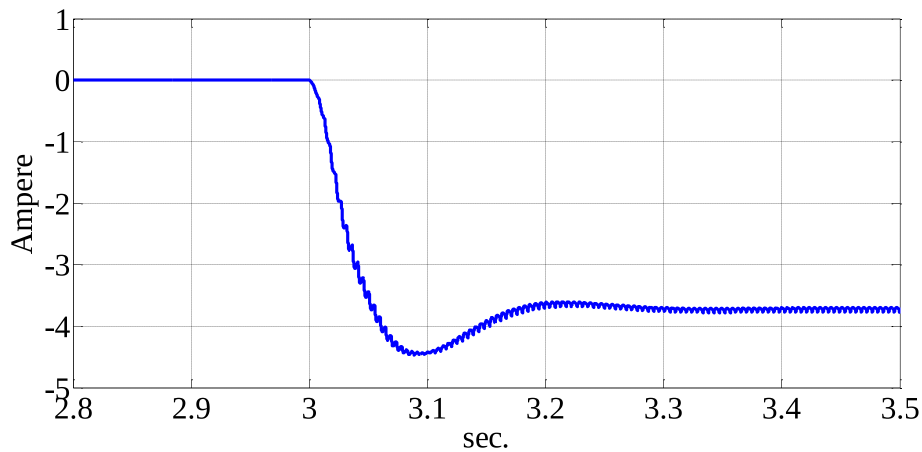
Figure 3.52: (a) The main component of the d axis current, (b) Starting moments, (c) Load changing moments.



(a)

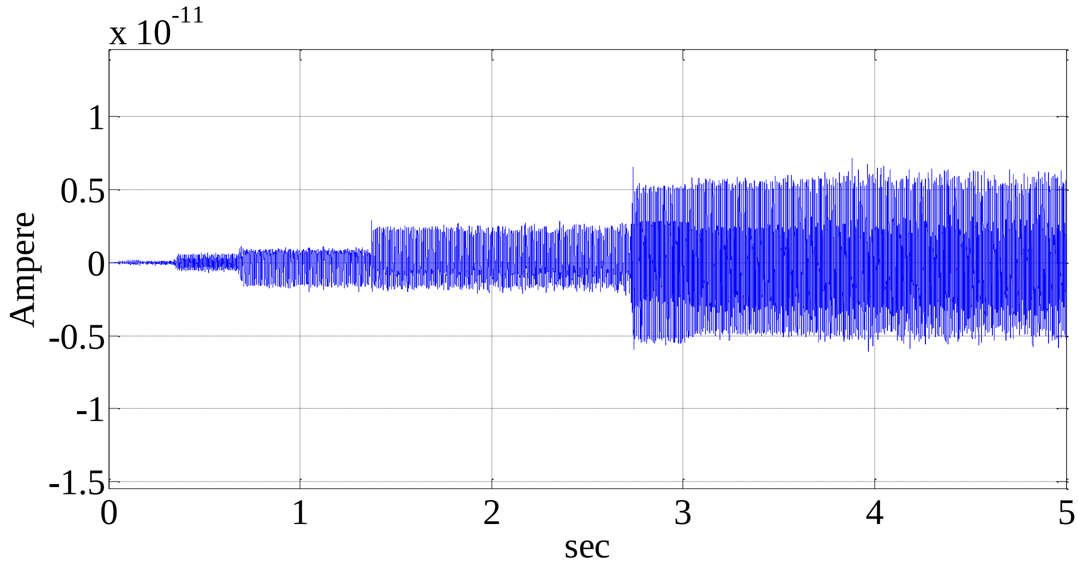


(b)

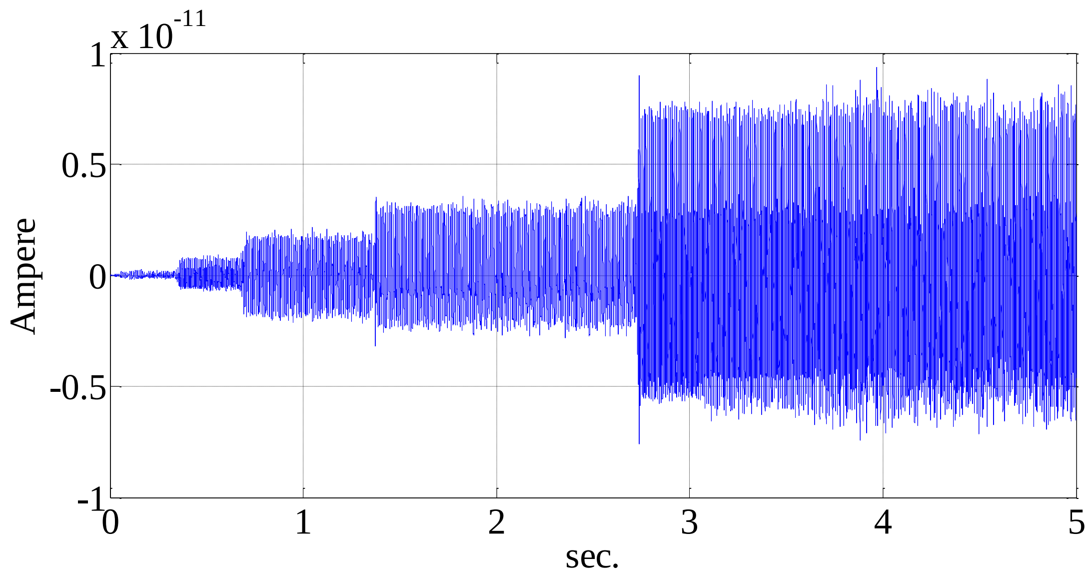


(c)

Figure 3.53: (a) The main component of the q axis current, (b) Starting moments, (c) Load changing moments.

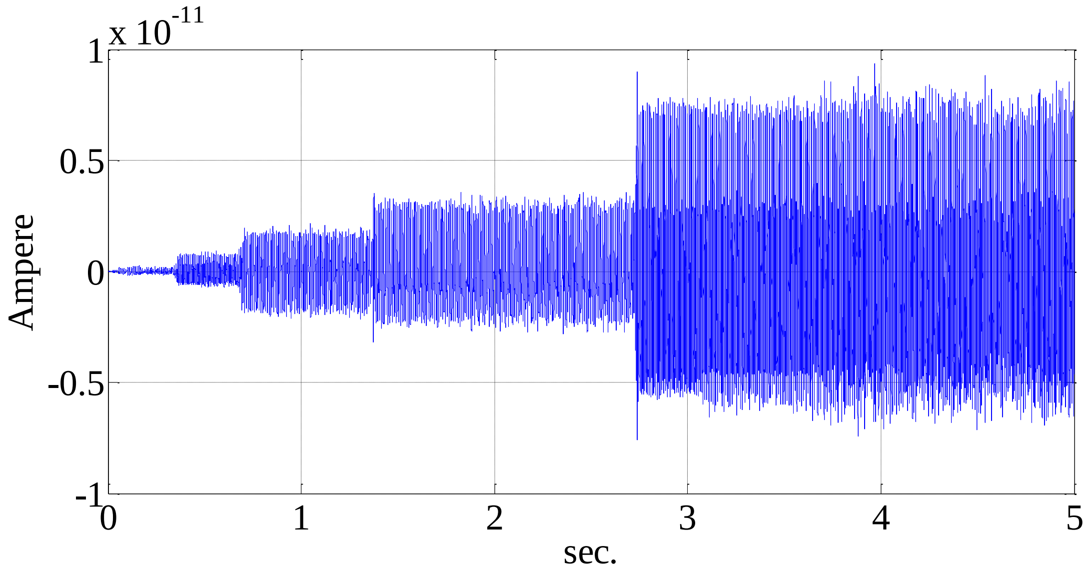


(a)

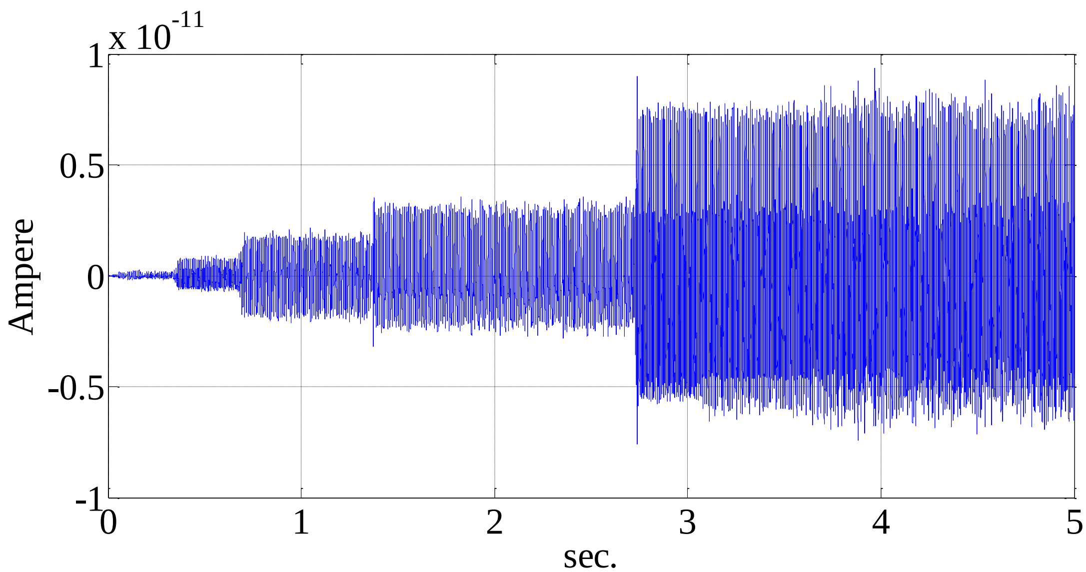


(b)

Figure 3.54: (a) The third component of the d axis current, (b) The third component of the q axis current.



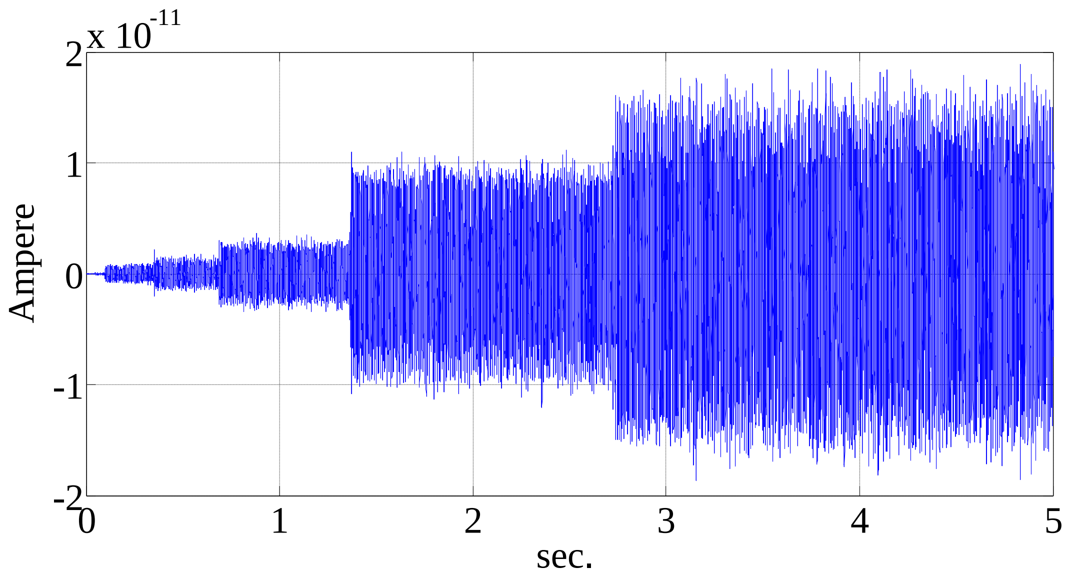
(a)



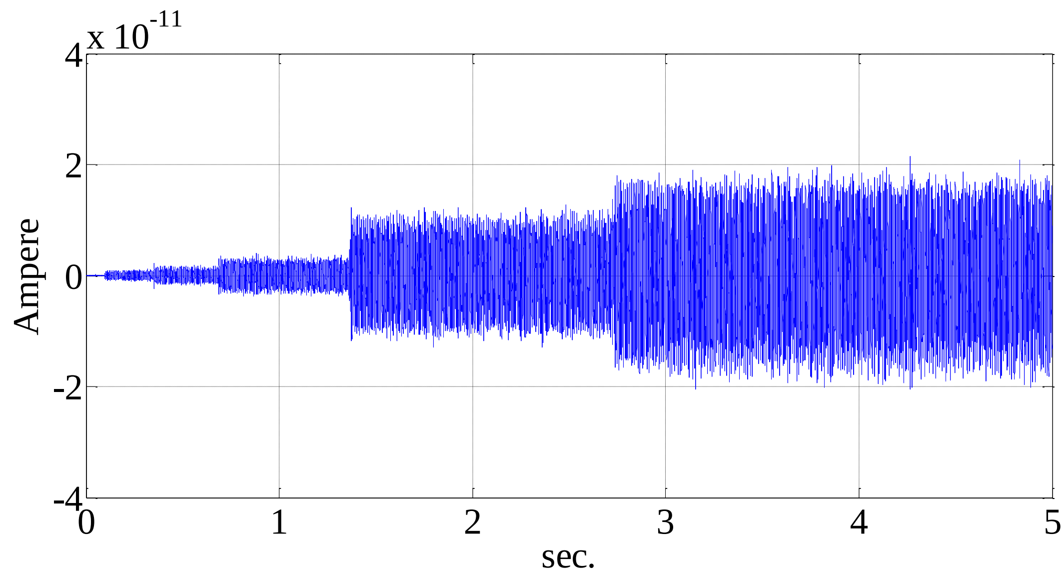
(b)

Figure 3.55: (a) The fifth component of the d axis current, (b) The fifth component of the q axis current.

Figures 3.52 to 3.56 show the different harmonics of the stator currents in the rotor reference frame. The main harmonics of the *d* and *q* axis currents are shown in the Figures 3.52 and 3.53. It can be seen that like the electromagnetic torque, the currents also react to the applied load torque and after passing the transients they settle down in their steady state values. The higher order harmonics of the stator currents have the magnitude of zero because there is no voltage applied to the higher order harmonics to excite them.



(a)



(b)

Figure 3.56: (a) The seventh component of the d axis current, (b) The seventh component of the q axis current.

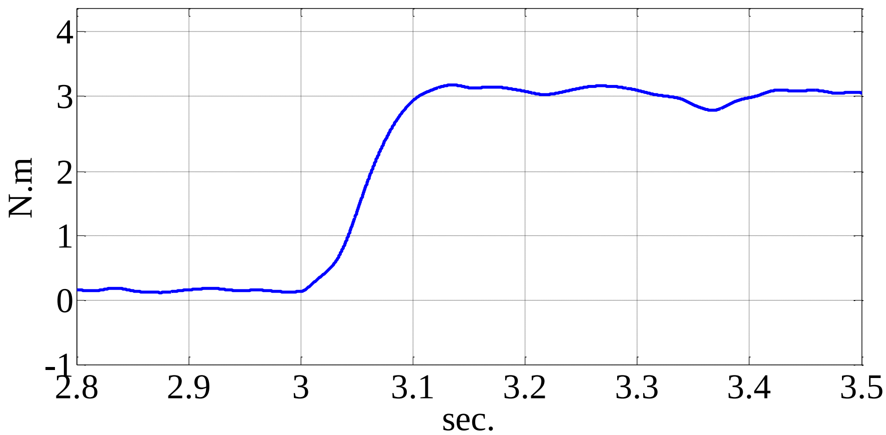


Figure 3.57: The electromagnetic torque of the machine (Experimental results).

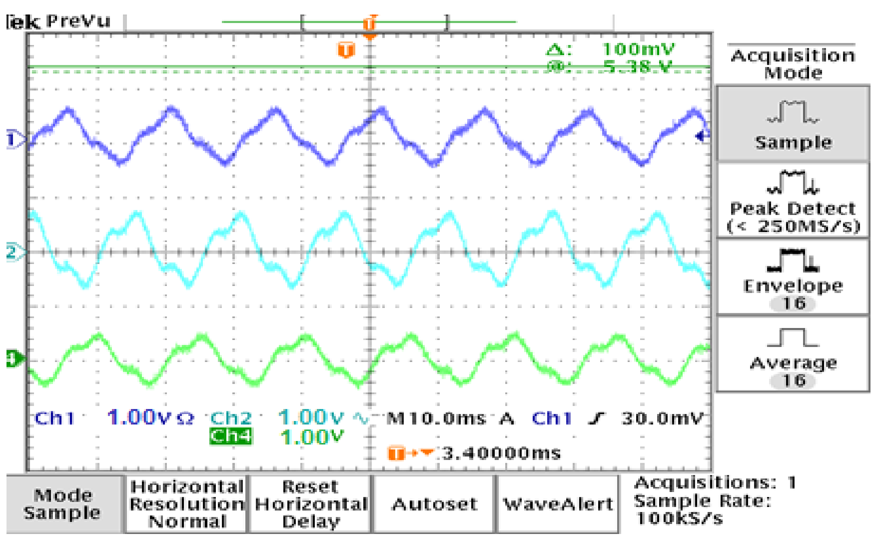


Figure 3.58: The currents of phases 'a', 'd' and 'g' before applying load (5A/scale).

3.4 Experimental Results of the Nine-Phase IPM Open Loop Run

In this section the nine-phase machine is tested using the DSP-FPGA controller in the prototype. The magnitude of the phase voltages is 120 (Volt) and the frequency is 60 (Hz). When the machine passes the transients and goes to the steady state, a mechanical load torque equal to 3

N.m (as shown in Figure 3.57) is applied to the machine. The phase current before and after applying load are shown in Figures 3.58 to 3.63.

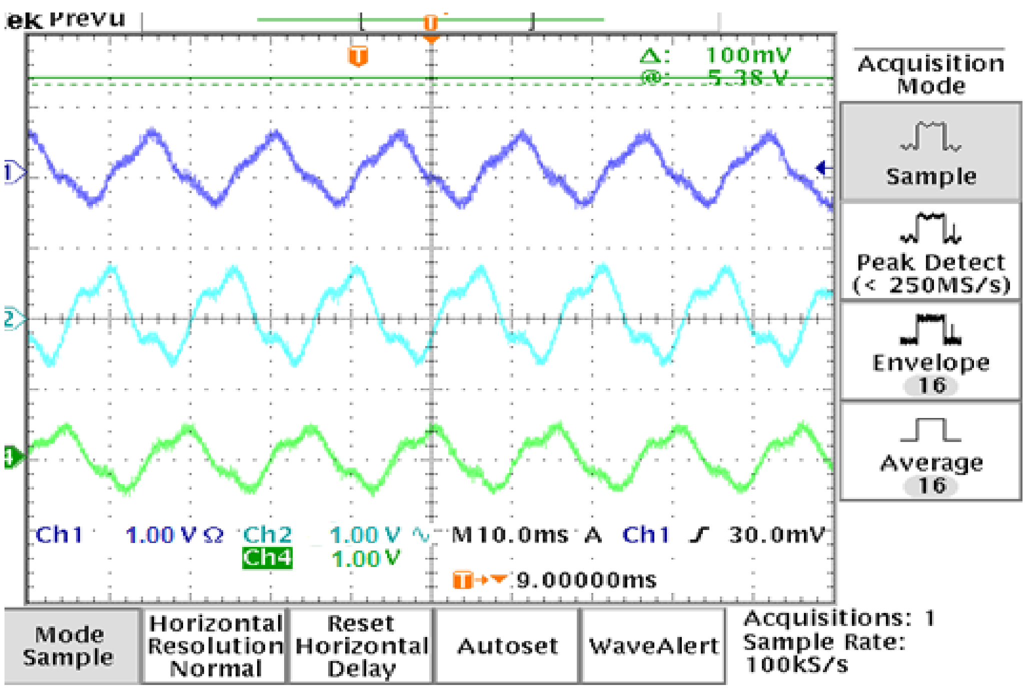


Figure 3.59: The currents of phases 'b', 'e' and 'h' before applying load (5A/scale).

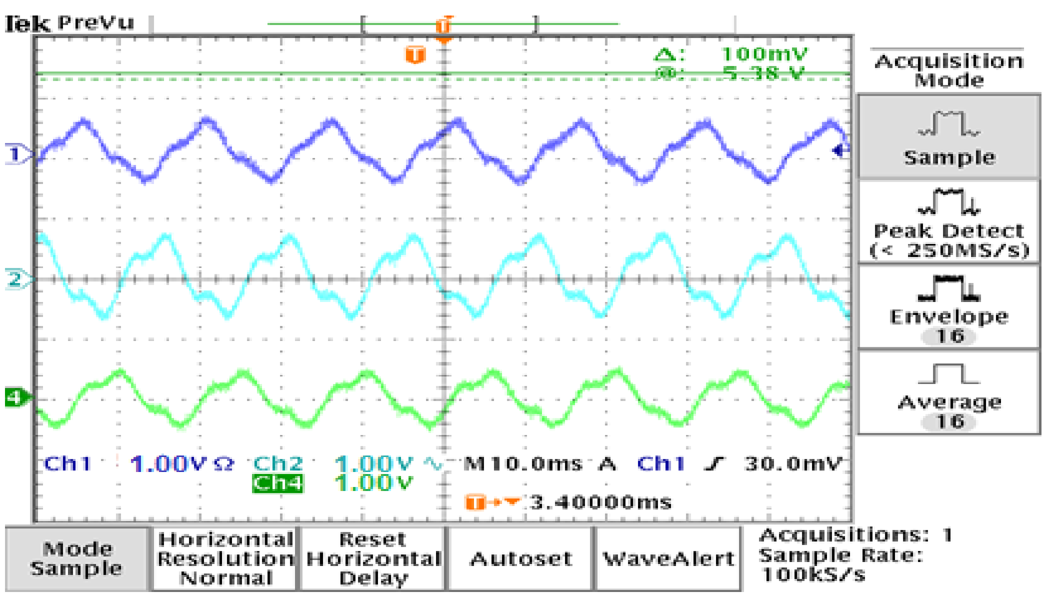


Figure 3.60: The currents of phases 'c', 'f' and 'i' before applying load (5 A/scale).

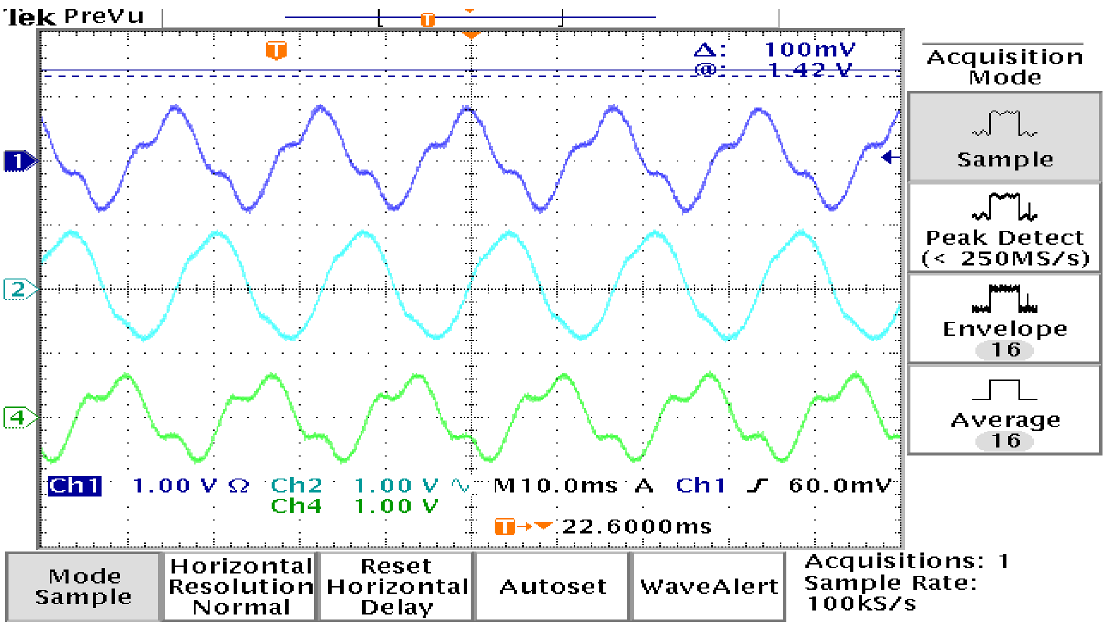


Figure 3.61: The currents of phases ‘a’, ‘d’ and ‘g’ after applying load (5 A/scale).

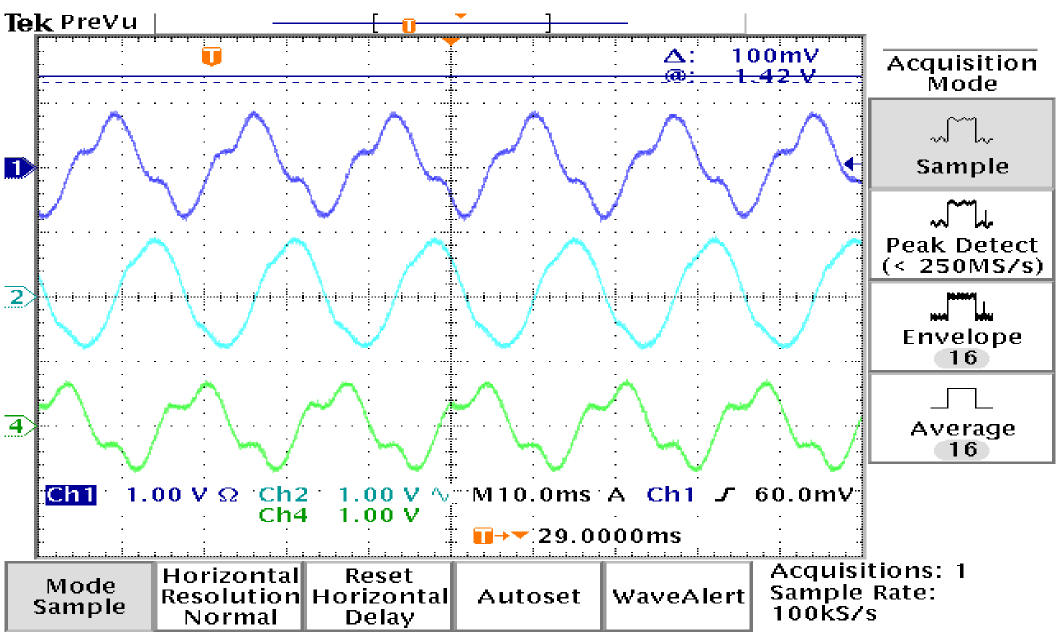


Figure 3.62: The currents of phases ‘b’, ‘e’ and ‘h’ after applying load (5 A/scale).

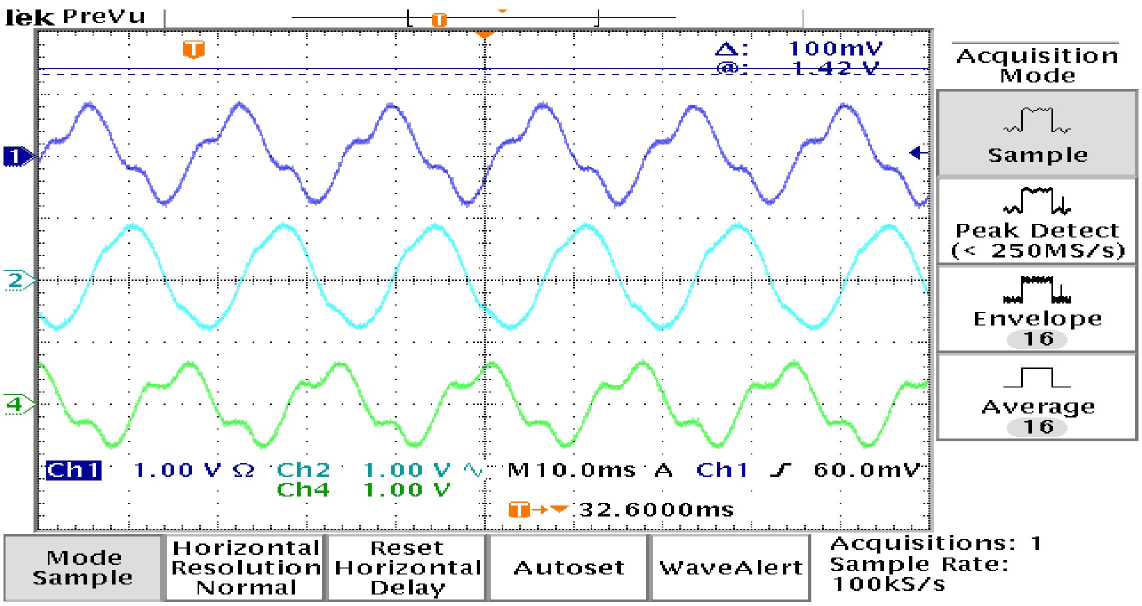
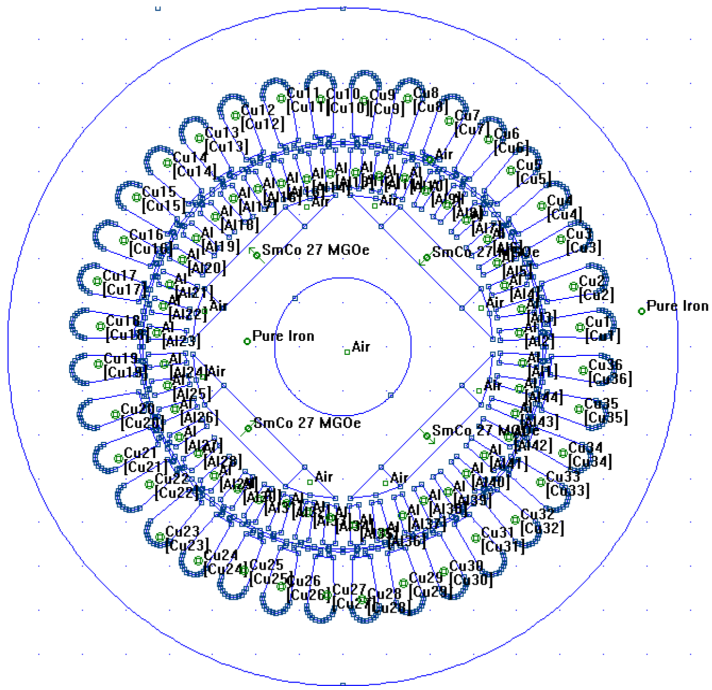


Figure 3.63: The currents of phases 'c', 'f' and 'i' after applying load (5 A/scale).

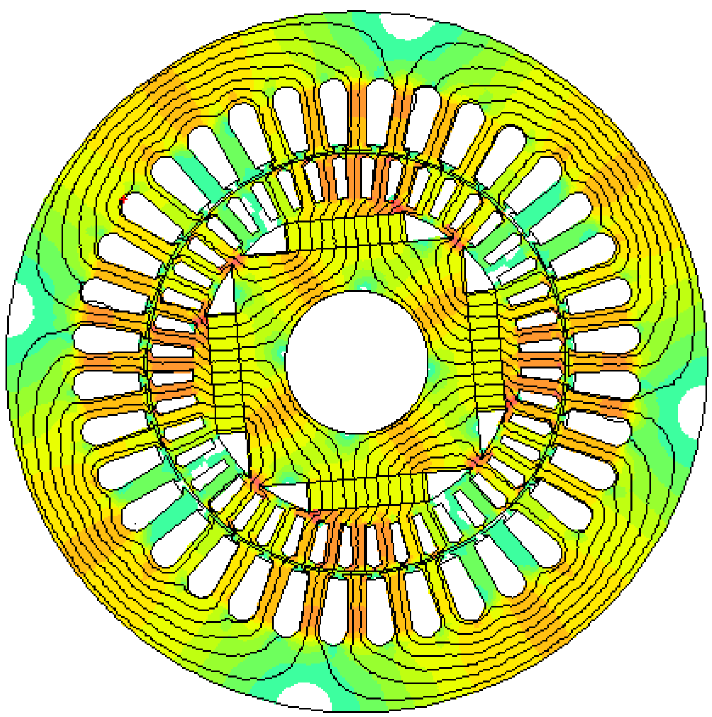
The current ripples can be seen in this figure The ripples are due to the variations of the stator inductances. The stator inductances variations are due to the rotor saliency of the machine.

3.5 Finite Element Analysis Using FEMM.

This section presents the steady state analysis of a Nine-Phase IPM machine in different operating conditions using FEMM (Finite Element Method Magnetics). In this study the machine inductances in different levels of stator currents are calculated. The first step is to generate the model of the machine in the FEMM. Figure 3.64 (a) shows the model of the machine generated in FEMM [83]. As it can be seen, this machine has 36 stator slots and 44 rotor bars also this machine has four magnets buried inside the rotor as the source of the magnetic flux linkage. To calculate the inductances of the d and q axis, the magnitude of the flux linkage, produced by the permanent magnets, needs to be known. To obtain this value, the model should be simulated while the stator currents are zero to obtain the pure flux linkage due to the permanent magnets.



(a)



(b)

Figure 3.64: (a) The machine model in FEMM, (b) The flux linkage due to the permanent magnets.

Figure 3.64 (b) shows the result of the simulation with zero stator currents. The flux linkage shown in the figure is due to the permanent magnet bars of the rotor of the machine. By plotting the airgap flux linkage, the magnitude of the flux linkage of the permanent magnet can be obtained.

Figure 3.65 shows the flux linkage of the machine in the airgap of the machine versus the circumferential angle of the airgap [83].

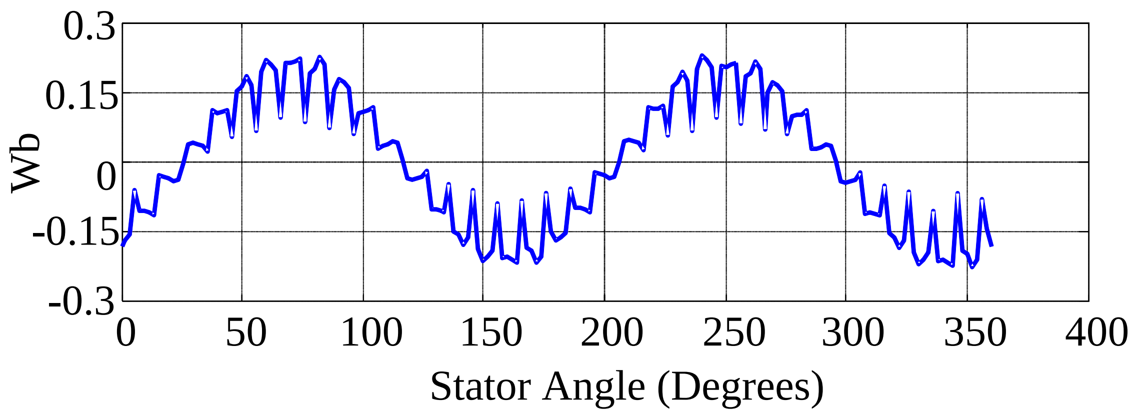


Figure 3.65: The flux linkage due to the permanent magnets in the airgap.

3.6 Computation of q and d Axis Inductances in Rotor Reference Frame

Due to the saturation of magnetic field path, variations of the permanent magnet flux linkage, the cross effect of the d and q axis flux linkages on each other the equivalent inductance which is seen from the stator of the machine vary with the variations of the stator currents. In most studies the machine inductances are considered constant which results in neglecting the effect of the variable parameters. To have a precise analysis of high performance drives, the exact value of the q and d axis inductances in each current level is needed. In this section the q and d inductances of the machine are calculated in different current magnitudes. After obtaining the value of the permanent magnet flux linkage which is 0.1807083000725188 (Wb), the different values of q and d axis inductances can be calculated according to the equations (3.37) and (3.38) respectively [43].

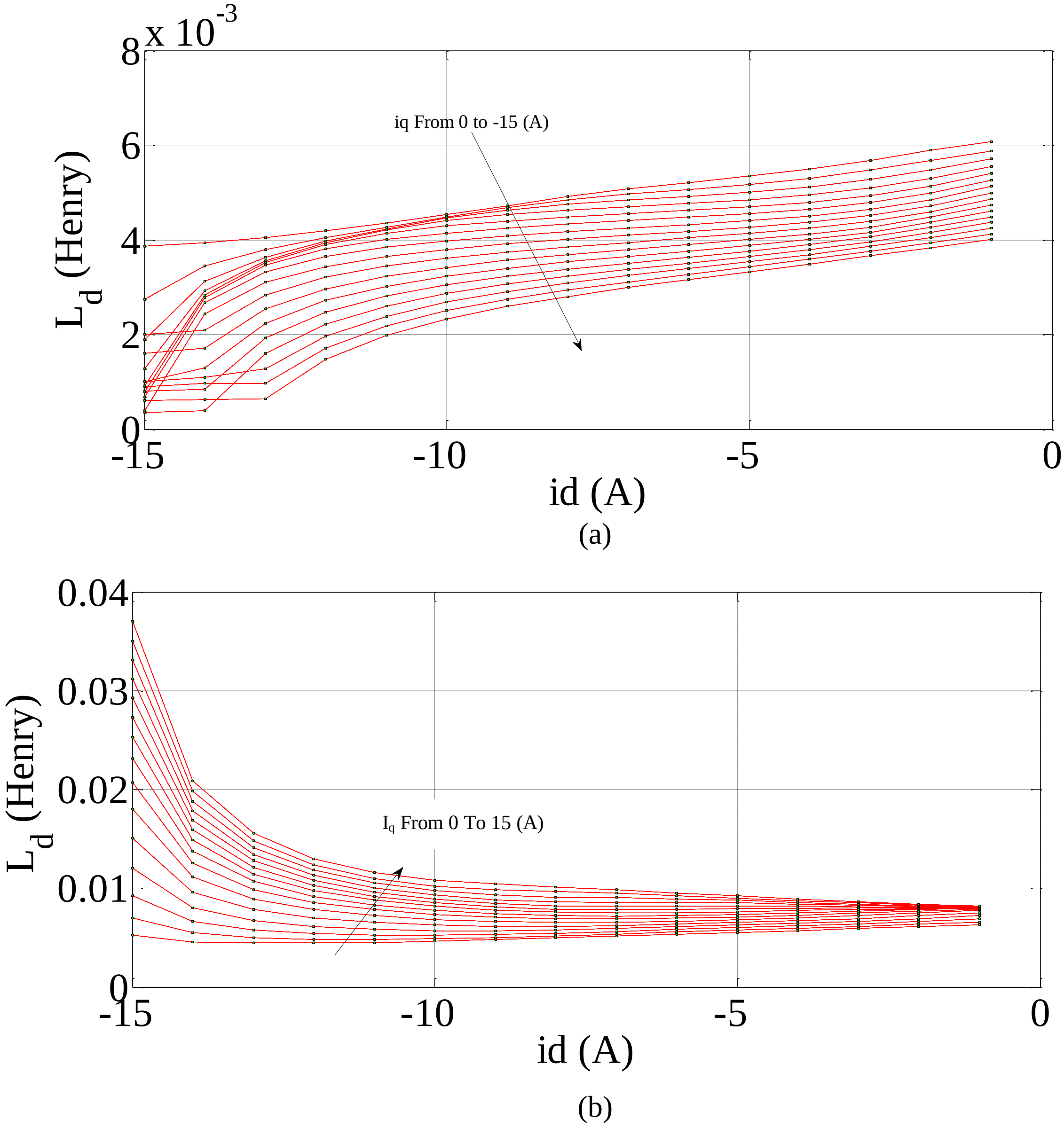


Figure 3.66: The Ld vs id < 0 when, (a) -15 < iq < 0, (b) 0 < iq < 15 (A).

$$L_q = \frac{\lambda_q}{i_q} \tag{3.37}$$

$$L_d = \frac{\lambda_d - \lambda_{pm}}{i_d} \tag{3.38}$$

Using equations (3.37) and (3.38) the inductances of the q and d can be calculated and plotted as Figures 3.66 to 3.69.

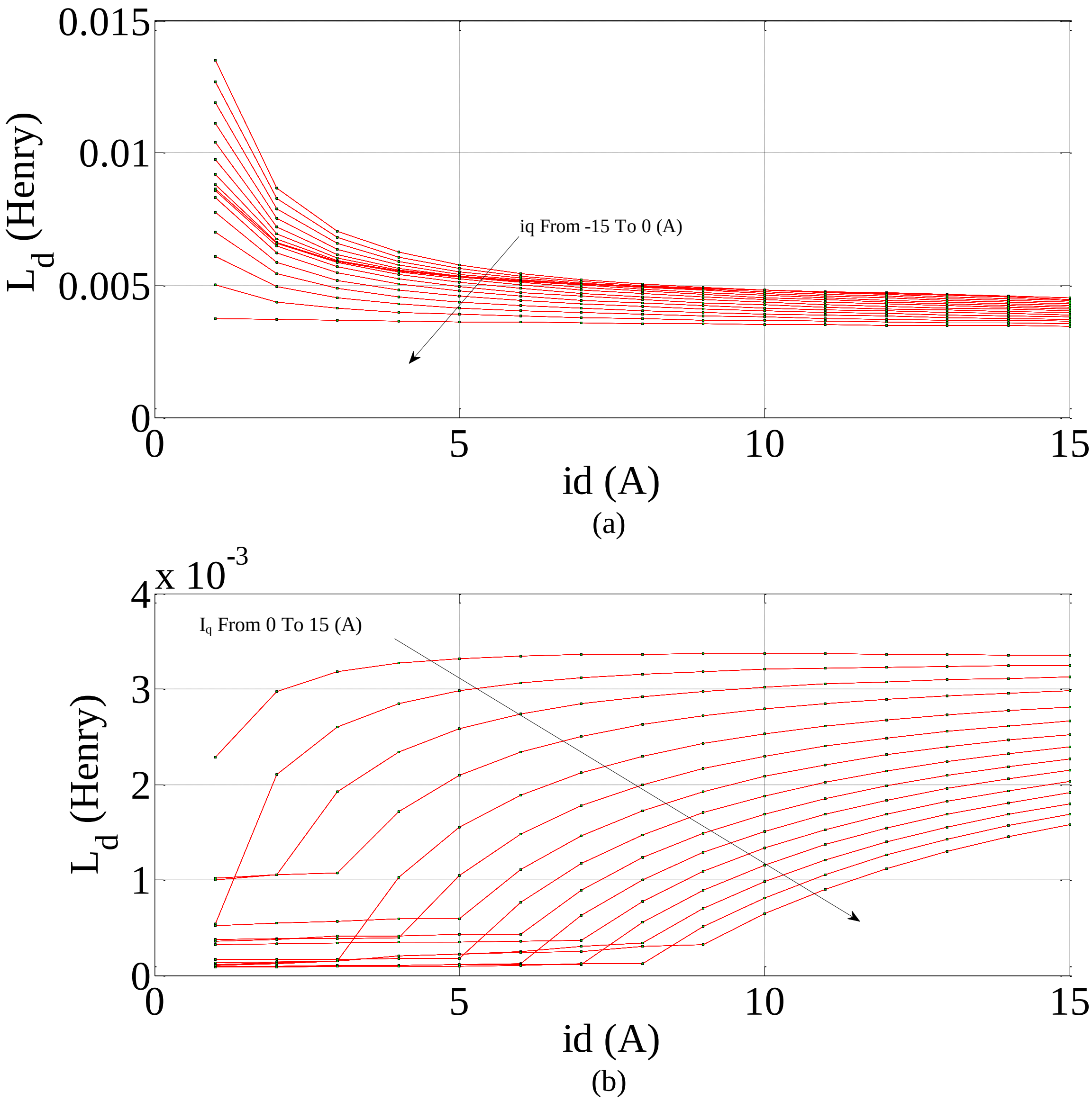
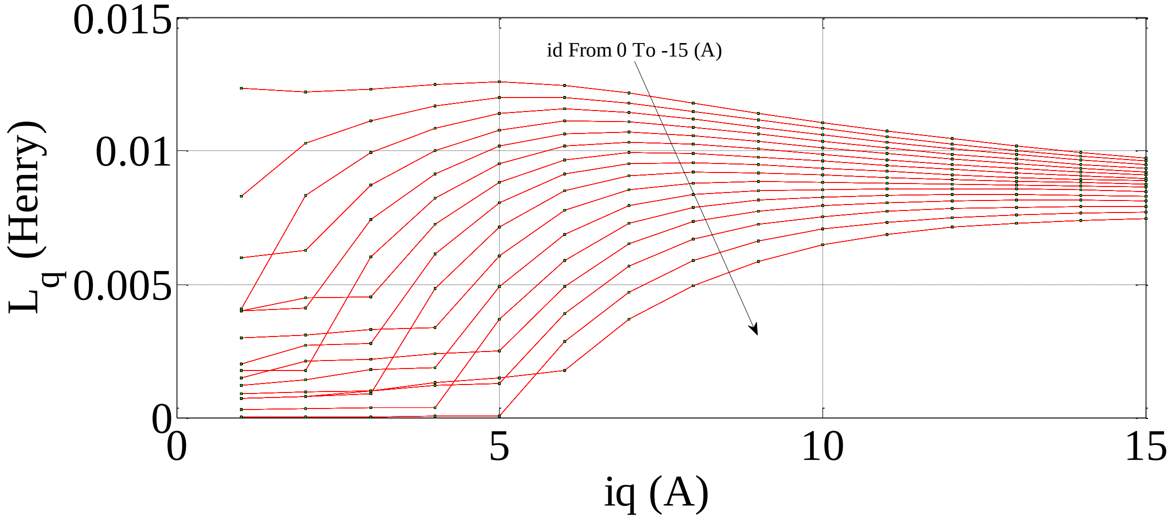
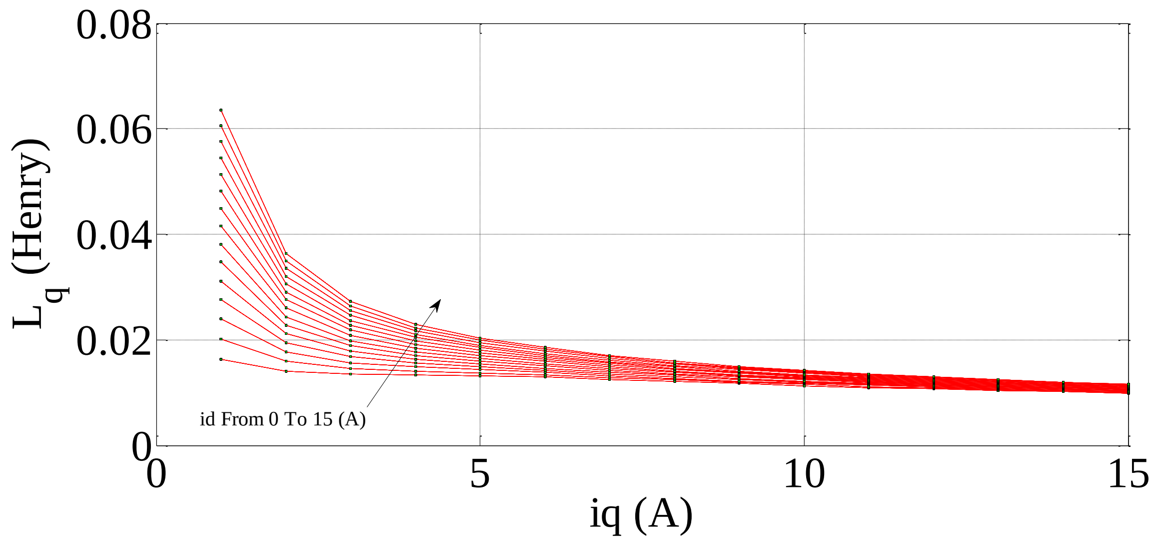


Figure 3.67: The Ld vs id >0 when, (a) -15<iq<0, (b) 0<iq<15 (A).

It should be noted that for convenience, in the equation (3.37) the variations of the flux linkage of permanent magnet (λ_{pm}) is neglected. Therefore, the effect of its variations will be included in the q and d inductances variations. Figure 3.66 shows the d axis inductance for negative d axis currents. In Figure 3.66 (A) the q axis current is negative. From Figure 3.66 (a) it can be seen that by increasing the currents of the d and q axis, the d axis inductance decreases. The saturation of the stator can obviously be seen from this figure. Figure 3.66 (b) shows the d axis inductance for positive values of the q axis currents. In this figure by increasing the q axis current, due to the cross coupling between the q and d axis, effective magnetic flux linkage decreases which reduces the stator saturation and consequently increase in d axis inductance. The d axis inductance for positive values of d axis currents are shown in Figure 3.67. Form Figure 3.67 (a) it can be seen that by increasing the q and d axis currents, the d axis inductance decreases which shows the saturation in the machine stator. In Figure 3.67 (b), by increasing the q axis current in positive region, the cross saturation causes the d inductance to decrease. By increasing the d axis current the effective magnetic flux linkage decreases which causes the d axis inductances to increase. Figure 3.68 shows the q axis inductances for positive values of q axis currents. From Figure 3.68 (a) by increasing the d axis current in negative parts, the inductances decrease (due to the saturation) and by increasing the q axis current, the effect of the d axis current on q axis inductance (through cross coupling) decreases, and the effective q axis inductance will increase. From Figure 3.68 (b) it can be seen that, by increasing the d axis current and by increasing the effective flux linkage of d axis, the q axis inductance decreases (due to saturation). Also by increasing the q axis current in positive region, the q axis inductance increases. Actually in this case when the q axis current increases in positive region, the positive flux linkage of q axis decreases the cross coupling effect of d axis in the q axis and reduces the stator saturation which can lead in increase of inductances. Figure 3.68 (b) shows the q axis inductance for positive values of q and d axis currents. By increasing the d axis currents, the effective flux linkage of the d axis decreases and reduces the saturation of the machine stator. The decrease in stator saturation causes increase in the q axis inductance.



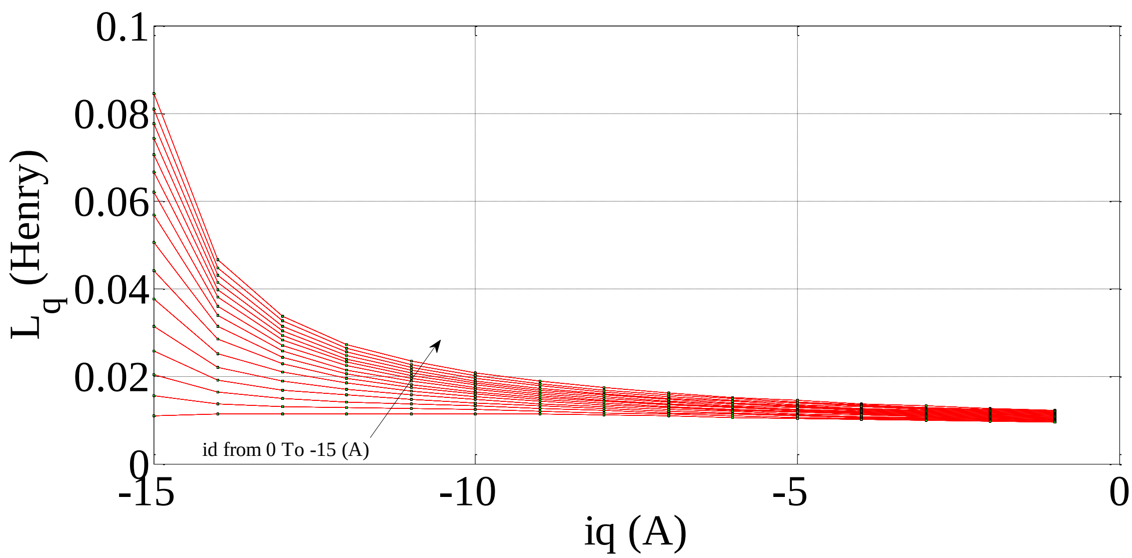
(a)



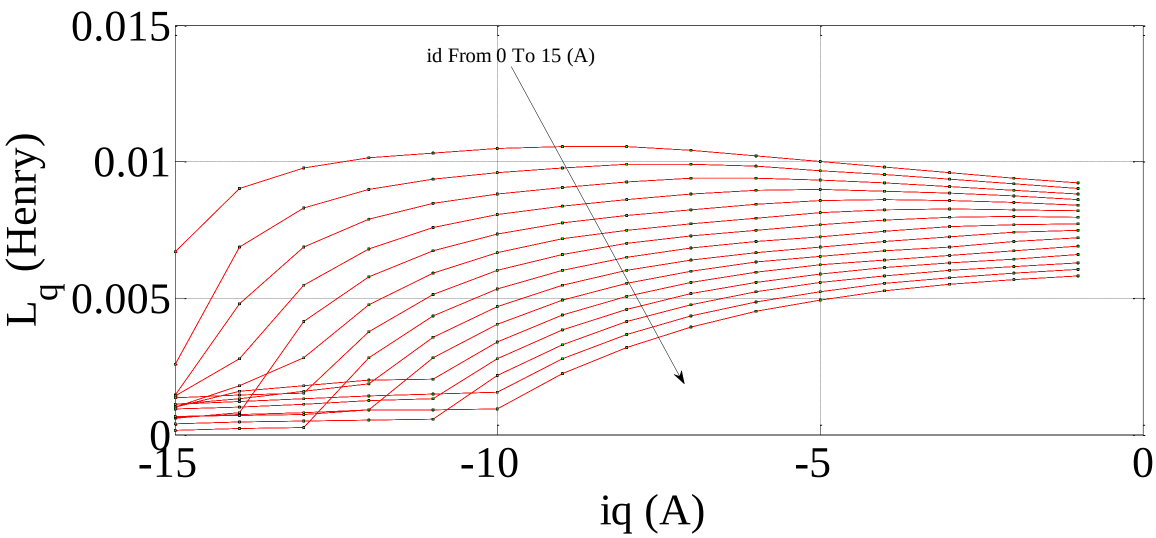
(b)

Figure 3.68: The Lq vs iq > 0 when, (a) -15 < id < 0, (b) 0 < id < 15 (A).

Figure 3.69 (a) shows the q axis inductances when d and q axis currents are negative. From this figure it can be seen that, by increasing the d axis current to the negative side, and due to the cross coupling the flux linkage of the q axis decreases and it reduces the saturation of the stator which consequently results in increase in the q axis inductance. Also by increasing the q axis current to the negative side when the d axis is negative the effective flux linkage of the q axis decreases which causes a decrease in the saturation and increase in q axis inductance.



(a)



(b)

Figure 3.69: The Lq vs iq < 0 when, (a) -15 < id < 0, (b) 0 < id < 15 (A).

Figure 3.69 (b) also shows the same inductance for positive values of d axis current. It can be seen that by increasing the d axis current and consequently d axis flux linkage, the stator inductance of q axis decreases. Increasing the q axis current to the negative values can saturate the machine stator and decreases the q axis inductance.

3.7 Coupled Modeling of the Symmetrical Triple Star IPM

3.7.1The Winding Design of the Machine

The nine-phase machine that was modeled in the section 3.2 has one single star point. To increase the reliability of the machine it is possible to connect it as a triple star machine, which basically means that the machine can be considered as three isolated sets of three phase machines. The machine windings can be designed according to the below procedure. The machine is composed of three sets of three phase machines. The 4-pole machine has 36 slots and each three phase set machine covers 12 slots. The slot angular pitch can be calculated as:

$$\gamma = \frac{180 \times P}{36} = \frac{180 \times 4}{36} = 20 \quad (Degree) \tag{3.39}$$

The slot between phases for each set and the full coil pitch can be calculated as:

$$SBPH = \frac{120}{\gamma} = \frac{120}{20} = 6, FCP = \frac{36}{P} = \frac{36}{4} = 9 \tag{3.40}$$

Since the machine has concentrated windings then the belt is equal to 1. And also since the machine is a symmetrical one the slot between two adjacent machines (SM) is [165]:

$$SM = \frac{6}{M} = 2 \tag{3.41}$$

Where 'M' is the number of the machines sets. For the machine 1 the winding scheme is:

Table 3.2 The winding scheme of machine 1.

$A1^+$	$A1^-$	$B1^+$	$B1^-$	$C1^+$	$C1^-$
1	10	7	16	13	22

$A1_-$	$A1_+$	$B1_-$	$B1_+$	$C1_-$	$C1_+$
10	19	16	25	22	31

$A1^+$	$A1^-$	$B1^+$	$B1^-$	$C1^+$	$C1^-$
19	28	25	34	31	4

$A1_-$	$A1_+$	$B1_-$	$B1_+$	$C1_-$	$C1_+$
28	1	34	7	4	13

The machine 2 has 2 slots shift from the machine 1, therefore the winding scheme for the machine 2 is:

Table 3.3 The winding scheme of machine 2.

$A2^+$	$A2^-$	$B2^+$	$B2^-$	$C2^+$	$C2^-$
3	12	9	18	15	24

$A2_-$	$A2_+$	$B2_-$	$B2_+$	$C2_-$	$C2_+$
12	21	18	27	24	33

$A2^+$	$A2^-$	$B2^+$	$B2^-$	$C2^+$	$C2^-$
21	30	27	36	33	6

$A2_-$	$A2_+$	$B2_-$	$B2_+$	$C2_-$	$C2_+$
30	3	36	9	6	15

The machine 3 has 2 slots shift from the machine 2, therefor the winding scheme for the machine 3 is:

Table 3.4 The winding scheme of machine 3.

$A3^+$	$A3^-$	$B3^+$	$B3^-$	$C3^+$	$C3^-$
5	14	11	20	17	26

$A3_-$	$A3_+$	$B3_-$	$B3_+$	$C3_-$	$C3_+$
14	23	20	29	26	35

$A3^+$	$A3^-$	$B3^+$	$B3^-$	$C3^+$	$C3^-$
23	32	29	4	35	8

$A3_-$	$A3_+$	$B3_-$	$B3_+$	$C3_-$	$C3_+$
32	5	2	11	8	17

Using the above tables, the clock diagram can be drawn as Figure 3.70 (a). In this new configuration if any of the machines has a fault, that machine can be removed by putting its stator voltage equal to zero and support the load or turbine using the rest of the machines. This feature simply means the machine can still support the load or absorb power from turbine in faulty condition.

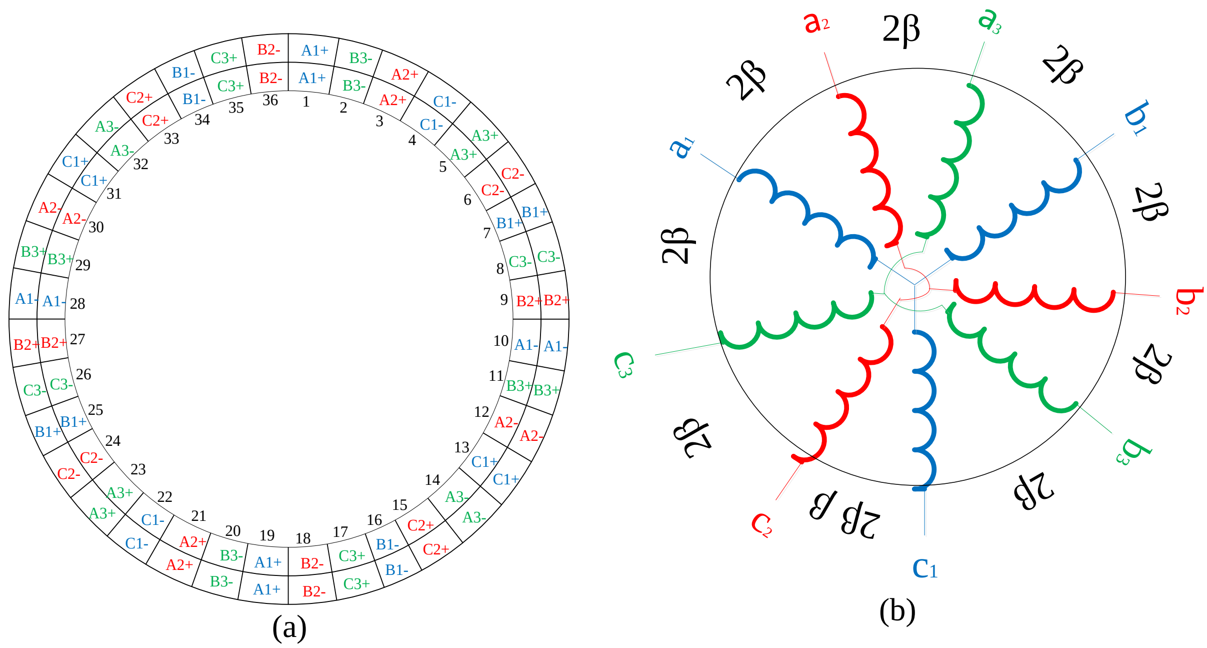


Figure 3.70: (a) The clock diagram of the symmetrical triple star machine, (b) The nine-phase IPM machine in symmetrical triple star connection.

3.7.2 The Model Equations

The machine modelling can be started from the voltage equations of the stator phases. Phase voltages for phases ‘a’, ‘b’, ‘c’ of each of the three phase IPM machines are given as equation (3.42). In this equation ‘I_{xi}’ is the current of phase ‘x’ and ‘$p\lambda_x$’ is the derivation of the flux linkage seen from the phase ‘x’ of the machine ‘i’ and the term ‘r_s’ also represents the stator resistance for each phase of the stator [83].

$$\begin{aligned} V_{ai} &= r_s i_{ai} + p\lambda_{ai} \\ V_{bi} &= r_s i_{bi} + p\lambda_{bi} \quad ,i=1,2,3 \\ V_{ci} &= r_s i_{ci} + p\lambda_{ci} \end{aligned} \tag{3.42}$$

The flux linkages of the machine phases can be represented as equation (3.43). In this set of equations, the terms 'L_{xixi}' represents the self-inductance of the phase *'x'* of the machine *'i'* and 'L_{xiyj}' represents the mutual inductance between phases 'x' of the machine *'i'* and phase *'y'* of the machine *'j'*. Also the term *'λ_{pmxi}'* represents the flux linkage due to the permanent magnets of the rotor seen from the phase *'x'* of the machine *'i'* [83].

$$
\begin{aligned}
\lambda_{a1} &= L_{a1a1}i_{a1} + L_{a1b1}i_{b1} + L_{a1c1}i_{c1} + L_{a1a2}i_{a2} + L_{a1b2}i_{b2} + L_{a1c2}i_{c2} + L_{a1a3}i_{a3} + L_{a1b3}i_{b3} + L_{a1c3}i_{c3} + \lambda_{pma1} \\
\lambda_{b1} &= L_{b1a1}i_{a1} + L_{b1b1}i_{b1} + L_{b1c1}i_{c1} + L_{b1a2}i_{a2} + L_{b1b2}i_{b2} + L_{b1c2}i_{c2} + L_{b1a3}i_{a3} + L_{b1b3}i_{b3} + L_{b1c3}i_{c3} + \lambda_{pmb1} \\
\lambda_{c1} &= L_{c1a1}i_{a1} + L_{c1b1}i_{b1} + L_{c1c1}i_{c1} + L_{c1a2}i_{a2} + L_{c1b2}i_{b2} + L_{c1c2}i_{c2} + L_{c1a3}i_{a3} + L_{c1b3}i_{b3} + L_{c1c3}i_{3} + \lambda_{pmc1} \\
\lambda_{a2} &= L_{a2a1}i_{a1} + L_{a2b1}i_{b1} + L_{a2c1}i_{c1} + L_{a2a2}i_{a2} + L_{a2b2}i_{b2} + L_{a2c2}i_{c2} + L_{a2a3}i_{a3} + L_{a2b3}i_{b3} + L_{a2c3}i_{c3} + \lambda_{pma2} \\
\lambda_{b2} &= L_{b2a1}i_{a1} + L_{b2b1}i_{b1} + L_{b2c1}i_{c1} + L_{b2a2}i_{a2} + L_{b2b2}i_{b2} + L_{b2c2}i_{c2} + L_{b2a3}i_{a3} + L_{b2b3}i_{b3} + L_{b2c3}i_{c3} + \lambda_{pmb2} \\
\lambda_{c2} &= L_{c2a1}i_{a1} + L_{c2b1}i_{b1} + L_{c2c1}i_{c1} + L_{c2a2}i_{a2} + L_{c2b2}i_{b2} + L_{c2c2}i_{c2} + L_{c2a3}i_{a3} + L_{c2b3}i_{b3} + L_{c2c3}i_{3} + \lambda_{pmc2} \\
\lambda_{a3} &= L_{a3a1}i_{a1} + L_{a3b1}i_{b1} + L_{a3c1}i_{c1} + L_{a3a2}i_{a2} + L_{a3b2}i_{b2} + L_{a3c2}i_{c2} + L_{a3a3}i_{a3} + L_{a3b3}i_{b3} + L_{a3c3}i_{c3} + \lambda_{pma3} \\
\lambda_{b3} &= L_{b3a1}i_{a1} + L_{b3b1}i_{b1} + L_{b3c1}i_{c1} + L_{b3a2}i_{a2} + L_{b3b2}i_{b2} + L_{b2c2}i_{c2} + L_{b3a3}i_{a3} + L_{b3b3}i_{b3} + L_{b2c3}i_{c3} + \lambda_{pmb3} \\
\lambda_{c3} &= L_{c3a1}i_{a1} + L_{c3b1}i_{b1} + L_{c3c1}i_{c1} + L_{c3a2}i_{a2} + L_{c3b2}i_{b2} + L_{c3c2}i_{c2} + L_{c3a3}i_{a3} + L_{c3b3}i_{b3} + L_{c3c3}i_{3} + \lambda_{pmc3}
\end{aligned}
\tag{3.43}
$$

The transformation matrix of equation (3.45) can be used to transform the voltage equations to the rotor reference frame. This transformation matrix has three parts. Each part is aligned to the phase ‘a’ of the corresponding machine. Therefore, the top part is aligned to the machine ‘1” the middle one is aligned with the machine ‘2’ and the bottom one is for machine ‘3’.

The flux linkages can be shown in the matrix form as:

$$\lambda_{abci} = L_s i_{sabci} + \lambda_{Pmabci} =$$

$$\begin{pmatrix} L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} & L_{a1a3} & L_{a1b3} & L_{a1c3} \\ L_{b1a1} & L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} & L_{b1a3} & L_{b1b3} & L_{b1c3} \\ L_{c1a1} & L_{c1b1} & L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} & L_{c1a3} & L_{c1b3} & L_{c1c3} \\ L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{a2a2} & L_{a2b2} & L_{a2c2} & L_{a2a3} & L_{a2b3} & L_{a2c3} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{b2b2} & L_{b2c2} & L_{b2a3} & L_{b2b3} & L_{b2c3} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{c2c2} & L_{c2a3} & L_{c2b3} & L_{c2c3} \\ L_{a3a1} & L_{a3b1} & L_{a3c1} & L_{a3a2} & L_{a3b2} & L_{a3c2} & L_{a3a3} & L_{a3b3} & L_{a3c3} \\ L_{b3a1} & L_{b3b1} & L_{b3c1} & L_{b3a2} & L_{b3b2} & L_{b3c2} & L_{b3a3} & L_{b3b3} & L_{b3c3} \\ L_{c3a1} & L_{c3b1} & L_{c3c1} & L_{c3a2} & L_{c3b2} & L_{c3c2} & L_{c3a3} & L_{c3b3} & L_{c3c3} \end{pmatrix} \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \\ i_{a3} \\ i_{b3} \\ i_{c3} \end{bmatrix} + \begin{bmatrix} \lambda_{pma1} \\ \lambda_{pmb1} \\ \lambda_{pmc1} \\ \lambda_{pma2} \\ \lambda_{pmb2} \\ \lambda_{pmc2} \\ \lambda_{pma3} \\ \lambda_{pmb3} \\ \lambda_{pmc3} \end{bmatrix} \quad (3.44)$$

$$T(\theta_r) =$$

$$\frac{2}{3}\begin{pmatrix} C(\theta_r+\beta) & C(\theta_r+\beta-\gamma) & C(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ S(\theta_r+\beta) & S(\theta_r+\beta-\gamma) & S(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r) & C(\theta_r-\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & S(\theta_r) & S(\theta_r-\gamma) & S(\theta_r+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\alpha) & C(\theta_r-\beta-\gamma) & C(\theta_r-\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r-\alpha) & S(\theta_r-\beta-\gamma) & S(\theta_r-\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad (3.45)$$

Where: $\gamma = 2\pi/3$, which is the phase shift between two adjacent phases, $\beta = 2\pi/9$ which is the phase shift between two adjacent machines and θ_r is the rotor angle. Also in this matrix ‘C’ represents ‘cos’ and ‘S’ represents ‘sin’.

The currents and flux linkages of equation (3.42) can be replaced by their corresponding currents in the rotor reference frame according to equation (3.46).

$$V_{xis} = r_{xis} T(\theta_r)^{-1} i_{qdor} + pT(\theta_r)^{-1} \lambda_{qdor} \tag{3.46}$$

Multiplying the $T(\theta_r)$ from the left side of the equation, results in:

$$T(\theta_r) V_{xis} = T(\theta_r) r_{xis} T(\theta_r)^{-1} i_{qdor} + T(\theta_r) pT(\theta_r)^{-1} \lambda_{qdor} \tag{3.47}$$

This can be rewritten as:

$$V_{qdor} = T(\theta_r) r_{xis} T(\theta_r)^{-1} i_{qdor} + T(\theta_r) pT(\theta_r)^{-1} \lambda_{qdor} \tag{3.48}$$

The different terms of the equation (3.48) can be represented as:

$$T(\theta_r) r_{xs} T(\theta_r)^{-1} i_{qdor} = \begin{pmatrix} r_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_s \end{pmatrix} \tag{3.49}$$

$$T(\theta_r) pT(\theta_r)^{-1} \lambda_{qdor} = T(\theta_r) pT(\theta_r)^{-1} \lambda_{qdor} + T(\theta_r) T(\theta_r)^{-1} p\lambda_{qdor} \tag{3.50}$$

In above equation the derivative parts can be written as:

$$T(\theta_r)\,pT(\theta_r)^{-1} =$$

$$\frac{2}{3}\begin{pmatrix} C(\theta_r+\beta) & C(\theta_r+\beta-\gamma) & C(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ S(\theta_r+\beta) & S(\theta_r+\beta-\gamma) & S(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r) & C(\theta_r-\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & S(\theta_r) & S(\theta_r-\gamma) & S(\theta_r+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\alpha) & C(\theta_r-\beta-\gamma) & C(\theta_r-\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r-\alpha) & S(\theta_r-\beta-\gamma) & S(\theta_r-\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\times \qquad (3.51)$$

$$p\begin{pmatrix} C(\theta_r+\beta) & S(\theta_r+\beta) & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ C(\theta_r+\beta-\gamma) & S(\theta_r+\beta-\gamma) & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ C(\theta_r+\beta+\gamma) & S(\theta_r+\beta+\gamma) & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r) & S(\theta_r) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r-\gamma) & S(\theta_r-\gamma) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r+\gamma) & S(\theta_r+\gamma) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\beta) & S(\theta_r-\beta) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\beta-\gamma) & S(\theta_r-\beta-\gamma) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\beta+\gamma) & S(\theta_r-\beta+\gamma) & 1 \end{pmatrix}$$

Substituting the derivative of the second matrix in to (3.51) results in:

$$T(\theta_r)pT(\theta_r)^{-1} =$$

$$\frac{2}{3}\begin{pmatrix} C(\theta_r+\beta) & C(\theta_r+\beta-\gamma) & C(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ S(\theta_r+\beta) & S(\theta_r+\beta-\gamma) & S(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r) & C(\theta_r-\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & S(\theta_r) & S(\theta_r-\gamma) & S(\theta_r+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\alpha) & C(\theta_r-\beta-\gamma) & C(\theta_r-\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r-\alpha) & S(\theta_r-\beta-\gamma) & S(\theta_r-\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \times \tag{3.52}$$

$$\omega_r\begin{pmatrix} -S(\theta_r+\beta) & C(\theta_r+\beta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -S(\theta_r+\beta-\gamma) & C(\theta_r+\beta-\gamma) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -S(\theta_r+\beta+\gamma) & C(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -S(\theta_r) & C(\theta_r) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -S(\theta_r-\gamma) & C(\theta_r-\gamma) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -S(\theta_r+\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -S(\theta_r-\beta) & C(\theta_r-\beta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -S(\theta_r-\beta-\gamma) & C(\theta_r-\beta-\gamma) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -S(\theta_r-\beta+\gamma) & C(\theta_r-\beta+\gamma) & 0 \end{pmatrix} = \omega_r\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

The flux linkage of the machine in the rotor reference frame can also be presented as below.

$$\lambda_{qdo} = T(\theta_r)L_{ss}i_{abci} + T(\theta_r)\lambda_{pm\_abci} \tag{3.53}$$

Substituting the currents by their corresponding currents in rotor reference frame results in:

$$\lambda_{qdo} = T(\theta_r)L_{ss}T^{-1}(\theta_r)i_{qdo} + T(\theta_r)\lambda_{pm\_abci} \tag{3.54}$$

The flux linkage due to the permanent magnet of the machine in the stator phases can be transformed to the rotor reference frame according to equation (3.55).

$$T(\theta_r)\lambda_{pm\_abci} = \frac{2}{3}\times$$

$$\begin{pmatrix} C(\theta_r+\beta) & C(\theta_r+\beta-\gamma) & C(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ S(\theta_r+\beta) & S(\theta_r+\beta-\gamma) & S(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r) & C(\theta_r-\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & S(\theta_r) & S(\theta_r-\gamma) & S(\theta_r+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\alpha) & C(\theta_r-\beta-\gamma) & C(\theta_r-\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r-\alpha) & S(\theta_r-\beta-\gamma) & S(\theta_r-\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad (3.55)$$

$$\times\begin{bmatrix} \lambda_{pm\_a1} \\ \lambda_{pm\_b1} \\ \lambda_{pm\_c1} \\ \lambda_{pm\_a2} \\ \lambda_{pm\_b2} \\ \lambda_{pm\_c2} \\ \lambda_{pm\_a3} \\ \lambda_{pm\_b3} \\ \lambda_{pm\_c3} \end{bmatrix} = \begin{bmatrix} \lambda_{pmq1r} \\ \lambda_{pmd1r} \\ \lambda_{pmor} \\ \lambda_{pmq2r} \\ \lambda_{pmd2r} \\ \lambda_{pmor} \\ \lambda_{pmq3r} \\ \lambda_{pmd3r} \\ \lambda_{pmor} \end{bmatrix}$$

The equation (3.54) also has a term which includes 'L_{ss}' representing the inductances of the stator of the machine. The inductances can be transformed to the rotor reference frame as equation (3.56) [83].

$$T(\theta_r)L_{ss}T(\theta_r)^{-1} =$$

$$\frac{2}{3}\begin{pmatrix} C(\theta_r+\beta) & C(\theta_r+\beta-\gamma) & C(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ S(\theta_r+\beta) & S(\theta_r+\beta-\gamma) & S(\theta_r+\beta+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r) & C(\theta_r-\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & S(\theta_r) & S(\theta_r-\gamma) & S(\theta_r+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\alpha) & C(\theta_r-\beta-\gamma) & C(\theta_r-\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r-\alpha) & S(\theta_r-\beta-\gamma) & S(\theta_r-\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \times \begin{pmatrix} L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} & L_{a1a3} & L_{a1b3} & L_{a1c3} \\ L_{b1a1} & L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} & L_{b1a3} & L_{b1b3} & L_{b1c3} \\ L_{c1a1} & L_{c1b1} & L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} & L_{c1a3} & L_{c1b3} & L_{c1c3} \\ L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{a2a2} & L_{a2b2} & L_{a2c2} & L_{a2a3} & L_{a2b3} & L_{a2c3} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{b2b2} & L_{b2c2} & L_{b2a3} & L_{b2b3} & L_{b2c3} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{c2c2} & L_{c2a3} & L_{c2b3} & L_{c2c3} \\ L_{a3a1} & L_{a3b1} & L_{a3c1} & L_{a3a2} & L_{a3b2} & L_{a3c2} & L_{a3a3} & L_{a3b3} & L_{a3c3} \\ L_{b3a1} & L_{b3b1} & L_{b3c1} & L_{b3a2} & L_{b3b2} & L_{b3c2} & L_{b3a3} & L_{b3b3} & L_{b3c3} \\ L_{c3a1} & L_{c3b1} & L_{c3c1} & L_{c3a2} & L_{c3b2} & L_{c3c2} & L_{c3a3} & L_{c3b3} & L_{c3c3} \end{pmatrix} \times$$

$$p\begin{pmatrix} C(\theta_r+\beta) & S(\theta_r+\beta) & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ C(\theta_r+\beta-\gamma) & S(\theta_r+\beta-\gamma) & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ C(\theta_r+\beta+\gamma) & S(\theta_r+\beta+\gamma) & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r) & S(\theta_r) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r-\gamma) & S(\theta_r-\gamma) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r+\gamma) & S(\theta_r+\gamma) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\beta) & S(\theta_r-\beta) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\beta-\gamma) & S(\theta_r-\beta-\gamma) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-\beta+\gamma) & S(\theta_r-\beta+\gamma) & 1 \end{pmatrix} = \tag{3.56}$$

$$\begin{pmatrix} L_{q1q1} & L_{q1d1} & L_{q101} & L_{q1q2} & L_{q1d2} & L_{q102} & L_{q1q3} & L_{q1d3} & L_{q103} \\ L_{d1q1} & L_{d1d1} & L_{d101} & L_{d1q2} & L_{d1d2} & L_{d102} & L_{d1q3} & L_{d1d3} & L_{d103} \\ L_{01q1} & L_{01d1} & L_{0101} & L_{01q2} & L_{01d2} & L_{0102} & L_{01q3} & L_{01d3} & L_{0103} \\ L_{q1q2} & L_{q1d2} & L_{q102} & L_{q2q2} & L_{q2d2} & L_{q202} & L_{q2q3} & L_{q2d3} & L_{q203} \\ L_{d1q2} & L_{d1d2} & L_{d102} & L_{d2q2} & L_{d2d2} & L_{d202} & L_{d2q3} & L_{d2d3} & L_{d203} \\ L_{01q2} & L_{01d2} & L_{0102} & L_{02q2} & L_{02d2} & L_{0202} & L_{02q3} & L_{02d3} & L_{0203} \\ L_{q1q3} & L_{q1d3} & L_{q103} & L_{q2q3} & L_{q2d3} & L_{q203} & L_{q3q3} & L_{q3d3} & L_{q303} \\ L_{d1q3} & L_{d1d3} & L_{d103} & L_{d2q3} & L_{d2d3} & L_{d203} & L_{d3q3} & L_{d3d3} & L_{d303} \\ L_{01q3} & L_{01d3} & L_{0103} & L_{02q3} & L_{02d3} & L_{0203} & L_{03q3} & L_{03d3} & L_{0303} \end{pmatrix}$$

Then using the permanent magnet flux linkages and the inductances of the machine in the rotor reference frame, the flux linkages of the machines can be represented in the rotor reference frame as equation (3.57).

$$
\begin{aligned}
\lambda_{q1} &= L_{q1q1}i_{q1} + L_{q1d1}i_{d1} + L_{q1o1}i_{o1} + L_{q1q2}i_{q2} + L_{q1d2}i_{d2} + L_{q1o2}i_{o2} + L_{q1q3}i_{q3} + L_{q1d3}i_{d3} + L_{q1o3}i_{o3} + \lambda_{pmq1} \\
\lambda_{d1} &= L_{d1d1}i_{d1} + L_{d1q1}i_{q1} + L_{d1o1}i_{o1} + L_{d1d2}i_{d2} + L_{d1q2}i_{q2} + L_{d1o2}i_{o2} + L_{d1d3}i_{d3} + L_{d1q3}i_{q3} + L_{d1o3}i_{o3} + \lambda_{pmd1} \\
\lambda_{o1} &= L_{o1d1}i_{d1} + L_{o1q1}i_{q1} + L_{o1o1}i_{o1} + L_{o1d2}i_{d2} + L_{o1q2}i_{q2} + L_{o1o2}i_{o2} + L_{o1d3}i_{d3} + L_{o1q3}i_{q3} + L_{o1o3}i_{o3} + \lambda_{pmo1} \\
\lambda_{q2} &= L_{q2q2}i_{q2} + L_{q2d2}i_{d2} + L_{q2o2}i_{o2} + L_{q2q1}i_{q1} + L_{q2d1}i_{d1} + L_{q2o1}i_{o1} + L_{q2q3}i_{q3} + L_{q2d3}i_{d3} + L_{q2o3}i_{o3} + \lambda_{pmq2} \\
\lambda_{d2} &= L_{d2d2}i_{d2} + L_{d2q2}i_{q2} + L_{d2o2}i_{o2} + L_{d2d1}i_{d1} + L_{d2q1}i_{q1} + L_{d2o1}i_{o1} + L_{d2d3}i_{d3} + L_{d2q3}i_{q3} + L_{d2o3}i_{o3} + \lambda_{pmd2} \\
\lambda_{o2} &= L_{o2d1}i_{d1} + L_{o2q1}i_{q1} + L_{o2o2}i_{o2} + L_{o2d2}i_{d2} + L_{o2q2}i_{q2} + L_{o2o2}i_{o2} + L_{o2d3}i_{d3} + L_{o2q3}i_{q3} + L_{o2o3}i_{o3} + \lambda_{pmo2} \\
\lambda_{q3} &= L_{q3q3}i_{q3} + L_{q3d3}i_{d3} + L_{q3o3}i_{o3} + L_{q3q1}i_{q1} + L_{q3d1}i_{d1} + L_{q3o1}i_{o1} + L_{q3q2}i_{q2} + L_{q3d2}i_{d2} + L_{q3o2}i_{o2} + \lambda_{pmq3} \\
\lambda_{d3} &= L_{d3d3}i_{d3} + L_{d3q3}i_{q3} + L_{d3o3}i_{o3} + L_{d3d1}i_{d1} + L_{d3q1}i_{q1} + L_{d3o1}i_{o1} + L_{d3d2}i_{d2} + L_{d3q2}i_{q2} + L_{q3o2}i_{o2} + \lambda_{pmd3} \\
\lambda_{o3} &= L_{o3d1}i_{d1} + L_{o3q1}i_{q1} + L_{o3o3}i_{o3} + L_{o3d2}i_{d2} + L_{o3q2}i_{q2} + L_{o3o2}i_{o2} + L_{o3d3}i_{d3} + L_{o3q3}i_{q3} + L_{o3o3}i_{o3} + \lambda_{pmo3}
\end{aligned}
\tag{3.57}
$$

Also the voltages and the currents of the machines can be transformed to the rotor reference frame as:

$$\begin{bmatrix} V_{q1} \\ V_{d1} \\ V_{o1} \\ V_{q2} \\ V_{d2} \\ V_{o2} \\ V_{q3} \\ V_{d3} \\ V_{o3} \end{bmatrix} = T(\theta_r)V_{abci} = T(\theta_r)\times \begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \\ V_{a2} \\ V_{b2} \\ V_{c2} \\ V_{a3} \\ V_{b3} \\ V_{c3} \end{bmatrix}, \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{o1} \\ i_{q2} \\ i_{d2} \\ i_{o2} \\ i_{q3} \\ i_{d3} \\ i_{o3} \end{bmatrix} = T(\theta_r)i_{abci} = T(\theta_r)\times \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \\ i_{a3} \\ i_{b3} \\ i_{c3} \end{bmatrix} \quad (3.58)$$

By separating the voltages of *q* and *d* axis of different machines, the voltage equations of the machines can be represented as [83]:

$$\begin{aligned} V_{q1} &= r_s i_{q1} + \omega_r \lambda_{d1} + p\lambda_{q1} \\ V_{d1} &= r_s i_{d1} - \omega_r \lambda_{q1} + p\lambda_{d1} \\ V_{o1} &= r_s i_{o1} + p\lambda_{o1} \end{aligned} \quad (3.59)$$

$$\begin{aligned} V_{q2} &= r_s i_{q2} + \omega_r \lambda_{d2} + p\lambda_{q2} \\ V_{d2} &= r_s i_{d2} - \omega_r \lambda_{q2} + p\lambda_{d2} \\ V_{o2} &= r_s i_{o2} + p\lambda_{o2} \end{aligned} \quad (3.60)$$

$$\begin{aligned} V_{q3} &= r_s i_{q3} + \omega_r \lambda_{d3} + p\lambda_{q3} \\ V_{d3} &= r_s i_{d3} - \omega_r \lambda_{q3} + p\lambda_{d3} \\ V_{o3} &= r_s i_{o3} + p\lambda_{o3} \end{aligned} \quad (3.61)$$

The generated electromagnetic torque of each machine can also be calculated using the co-energy equation. The co-energy of the machine 1 can be presented as a function of the stator currents and the flux linkages as [152]:

$$W_{co1} = \frac{1}{2} i_{abc1}{}^{t} L_{ss} i_{abc1,2,3} + i_{s1}{}^{t} \lambda_{pm\_1} \tag{3.62}$$

From the co-energy equation, the electromagnetic torque can be derived as:

$$T_{e1} = \frac{\partial W_{co1}}{\partial \theta_{rm}} \tag{3.63}$$

The equation (3.63) is equal to the equation (3.64).

$$T_{e1} = \frac{1}{2} i_{abc1}{}^{t} \frac{\partial L_{ss}}{\partial \theta_{rm}} i_{abc1,2,3} + i_{abc1}{}^{t} \frac{\partial \lambda_{pm\_1}}{\partial \theta_{rm}} \tag{3.64}$$

Since the previous equations are in term of the electrical angle, the mechanical angle of the equation (3.64) needs to be converted to the electrical equation as equation (3.65)

$$\theta_{r} = \frac{P}{2} \theta_{rm} \tag{3.65}$$

Therefore, the torque equation changes to equation (3.66).

$$T_{e1} = \frac{P}{2} \frac{1}{2} i_{abc1}{}^{t} \frac{\partial L_{ss}}{\partial \theta_{r}} i_{abc1,2,3} + \frac{P}{2} i_{s1}{}^{t} \frac{\partial \lambda_{pm\_1}}{\partial \theta_{r}} \tag{3.66}$$

Substituting the stator currents with their corresponding values in rotor reference frame results in:

$$T_{e1} = \frac{3}{2}\frac{P}{2}(i_{qdo1})^t \, T(\theta_r)\frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} i_{qdo1,2,3} + \frac{3}{2}\frac{P}{2}(i_{qdo1})^t \, T(\theta_r)\frac{\partial \lambda_{pm\_1}}{\partial \theta_r} \tag{3.67}$$

The equation can be rewritten as:

$$T_{e1} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^t T(\theta_r)\frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^t T(\theta_r)\frac{\partial \lambda_{pm}}{\partial \theta_r} \tag{3.68}$$

The equation (3.68) simplifies to equation (3.69).

$$T_{e1} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^t L_{qdo} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t\,t} \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix} \tag{3.69}$$

Replacing the inductance matrix from the equation (3.56) results in:

$$T_{e1} =$$

$$\frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{pmatrix} L_{q11} & L_{q1d1} & L_{q101} & L_{q1q2} & L_{q1d2} & L_{q102} & L_{q1q3} & L_{q1d3} & L_{q103} \\ L_{d1q1} & L_{d11} & L_{d101} & L_{d1q2} & L_{d1d2} & L_{d102} & L_{d1q3} & L_{d1d3} & L_{d103} \\ L_{01q1} & L_{01d1} & L_{0101} & L_{01q2} & L_{01d2} & L_{0102} & L_{01q3} & L_{01d3} & L_{0103} \\ L_{q1q2} & L_{q1d2} & L_{q102} & L_{q22} & L_{q2d2} & L_{q202} & L_{q2q3} & L_{q2d3} & L_{q203} \\ L_{d1q2} & L_{d1d2} & L_{d102} & L_{d2q2} & L_{d22} & L_{d202} & L_{d2q3} & L_{d2d3} & L_{d203} \\ L_{01q2} & L_{01d2} & L_{0102} & L_{02q2} & L_{02d2} & L_{0202} & L_{02q3} & L_{02d3} & L_{0203} \\ L_{q1q3} & L_{q1d3} & L_{q103} & L_{q2q3} & L_{q2d3} & L_{q203} & L_{q33} & L_{q3d3} & L_{q303} \\ L_{d1q3} & L_{d1d3} & L_{d103} & L_{d2q3} & L_{d2d3} & L_{d203} & L_{d3q3} & L_{d33} & L_{d303} \\ L_{01q3} & L_{01d3} & L_{0103} & L_{02q3} & L_{02d3} & L_{0203} & L_{03q3} & L_{03d3} & L_{0303} \end{pmatrix} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \quad (3.70)$$

$$\frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix}$$

This equation is equal to:

$$
T_{e1} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} \begin{pmatrix}
L_{q11}i_{q1} + L_{q1d1}i_{d1} + L_{q101}i_{01} + L_{q1q2}i_{q2} + L_{q1d2}i_{d2} + L_{q102}i_{02} + L_{q1q3}i_{q3} + L_{q1d3}i_{d3} + L_{q103}i_{03} \\
L_{d1q1}i_{q1} + L_{d11}i_{d1} + L_{d101}i_{01} + L_{d1q2}i_{q2} + L_{d1d2}i_{d2} + L_{d102}i_{02} + L_{d1q3}i_{q3} + L_{d1d3}i_{d3} + L_{d103}i_{03} \\
L_{01q1}i_{q1} + L_{01d1}i_{d1} + L_{0101}i_{01} + L_{01q2}i_{q2} + L_{01d2}i_{d2} + L_{0102}i_{02} + L_{01q3}i_{q3} + L_{01d3}i_{d3} + L_{0103}i_{03} \\
L_{q1q2}i_{q1} + L_{q1d2}i_{d1} + L_{q102}i_{01} + L_{q22}i_{q2} + L_{q2d2}i_{d2} + L_{q202}i_{02} + L_{q2q3}i_{q3} + L_{q2d3}i_{d3} + L_{q203}i_{03} \\
L_{d1q2}i_{q1} + L_{d1d2}i_{d1} + L_{d102}i_{01} + L_{d2q2}i_{q2} + L_{d22}i_{d2} + L_{d202}i_{02} + L_{d2q3}i_{q3} + L_{d2d3}i_{d3} + L_{d203}i_{03} \\
L_{01q2}i_{q1} + L_{01d2}i_{d1} + L_{0102}i_{01} + L_{02q2}i_{q2} + L_{02d2}i_{d2} + L_{0202}i_{02} + L_{02q3}i_{q3} + L_{02d3}i_{d3} + L_{0203}i_{03} \\
L_{q1q3}i_{q1} + L_{q1d3}i_{d1} + L_{q103}i_{01} + L_{q2q3}i_{q2} + L_{q2d3}i_{d2} + L_{q203}i_{02} + L_{q3q3}i_{q3} + L_{q3d3}i_{d3} + L_{q303}i_{03} \\
L_{d1q3}i_{q1} + L_{d1d3}i_{d1} + L_{d103}i_{01} + L_{d2q3}i_{q2} + L_{d2d3}i_{d2} + L_{d203}i_{02} + L_{d3q3}i_{q3} + L_{d3d3}i_{d3} + L_{d303}i_{03} \\
L_{01q3}i_{q1} + L_{01d3}i_{d1} + L_{0103}i_{01} + L_{02q3}i_{q2} + L_{02d2}i_{d2} + L_{0203}i_{02} + L_{03q3}i_{q3} + L_{03d3}i_{d3} + L_{0303}i_{03}
\end{pmatrix}
$$

$$
+ \frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix} \tag{3.71}
$$

The non-zero terms of the equation (3.71) after simplifying are:

$$T_{e1} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} \begin{pmatrix} L_{q11}i_{q1} + L_{q1d1}i_{d1} + L_{q101}i_{01} + L_{q1q2}i_{q2} + L_{q1d2}i_{d2} + \\ L_{q102}i_{02} + L_{q1q3}i_{q3} + L_{q1d3}i_{d3} + L_{q103}i_{03} \end{pmatrix} i_{d1} - \\ \begin{pmatrix} L_{d1q1}i_{q1} + L_{d11}i_{d1} + L_{d101}i_{01} + L_{d1q2}i_{q2} + L_{d1d2}i_{d2} + \\ L_{d102}i_{02} + L_{d1q3}i_{q3} + L_{d1d3}i_{d3} + L_{d103}i_{03} \end{pmatrix} i_{q1} + \\ \begin{pmatrix} L_{01q1}i_{q1} + L_{01d1}i_{d1} + L_{0101}i_{01} + L_{01q2}i_{q2} + L_{01d2}i_{d2} + \\ L_{0102}i_{02} + L_{01q3}i_{q3} + L_{01d3}i_{d3} + L_{0103}i_{03} \end{pmatrix} i_{01} \end{bmatrix} \quad (3.72)$$

$$+ \frac{3}{2}\frac{P}{2}\left(\lambda_{pmq1}i_{d1} + \lambda_{pmd1}i_{q1} + \lambda_{pm01}i_{01}\right)$$

The co-energy of the machine 2 also can be presented as function of the stator currents and the flux linkages as:

$$W_{co2} = \frac{1}{2} i_{abco2}{}^{t} L_{ss} i_{abc1,2,3} + i_{s2}{}^{t} \lambda_{pm\_2} \quad (3.73)$$

From the torque co-energy equation, the electromagnetic torque can be derived as:

$$T_{e2} = \frac{\partial W_{co2}}{\partial \theta_{rm}} \quad (3.74)$$

The equation (3.74) is equal to the below one.

$$T_{e2} = \frac{1}{2} i_{abc2}{}^{t} \frac{\partial L_{ss}}{\partial \theta_{rm}} i_{abc1,2,3} + i_{abc2}{}^{t} \frac{\partial \lambda_{pm\_2}}{\partial \theta_{rm}} \quad (3.75)$$

Again the mechanical angle of the equation (3.75) needs to be converted to the electrical angle as presented in equation (3.76):

$$\theta_r = \frac{P}{2}\theta_{rm} \quad (3.76)$$

Therefore, the torque equation changes to equation (3.77):

$$T_{e2} = \frac{P}{2}\frac{1}{2} i_{abc2}{}^{t} \frac{\partial L_{ss}}{\partial \theta_r} i_{abc1,2,3} + \frac{P}{2} i_{s2}{}^{t} \frac{\partial \lambda_{pm\_2}}{\partial \theta_r} \tag{3.77}$$

Substituting the stator currents with their corresponding values in the rotor reference frame results in:

$$T_{e2} = \frac{3}{2}\frac{P}{2}(i_{qdo2})^{t}\, T(\theta_r)\frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} i_{qdo1,2,3} + \frac{3}{2}\frac{P}{2}(i_{qdo2})^{t}\, T(\theta_r)\frac{\partial \lambda_{pm\_2}}{\partial \theta_r} \tag{3.78}$$

The equation can be rewritten as:

$$T_{e2} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} T(\theta_r)\frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} T(\theta_r)\frac{\partial \lambda_{pm}}{\partial \theta_r} \tag{3.79}$$

The equation (3.79) simplifies to equation (3.80).

$$T_{e2} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} L_{qdo} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix} \tag{3.80}$$

Replacing the qd inductance matrix in to the equation (3.80) results in:

$$
T_{e2} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} \left(\begin{matrix}
L_{q11} & L_{q1d1} & L_{q101} & L_{q1q2} & L_{q1d2} & L_{q102} & L_{q1q3} & L_{q1d3} & L_{q103} \\
L_{d1q1} & L_{d11} & L_{d101} & L_{d1q2} & L_{d1d2} & L_{d102} & L_{d1q3} & L_{d1d3} & L_{d103} \\
L_{01q1} & L_{01d1} & L_{0101} & L_{01q2} & L_{01d2} & L_{0102} & L_{01q3} & L_{01d3} & L_{0103} \\
L_{q1q2} & L_{q1d2} & L_{q102} & L_{q22} & L_{q2d2} & L_{q202} & L_{q2q3} & L_{q2d3} & L_{q203} \\
L_{d1q2} & L_{d1d2} & L_{d102} & L_{d2q2} & L_{d22} & L_{d202} & L_{d2q3} & L_{d2d3} & L_{d203} \\
L_{01q2} & L_{01d2} & L_{0102} & L_{02q2} & L_{02d2} & L_{0202} & L_{02q3} & L_{02d3} & L_{0203} \\
L_{q1q3} & L_{q1d3} & L_{q103} & L_{q2q3} & L_{q2d3} & L_{q203} & L_{q33} & L_{q3d3} & L_{q303} \\
L_{d1q3} & L_{d1d3} & L_{d103} & L_{d2q3} & L_{d2d3} & L_{d203} & L_{d3q3} & L_{d33} & L_{d303} \\
L_{01q3} & L_{01d3} & L_{0103} & L_{02q3} & L_{02d3} & L_{0203} & L_{03q3} & L_{03d3} & L_{0303}
\end{matrix} \right) \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \qquad (3.81)
$$

$$
\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix}
$$

The equation (3.81) is equal to equation (3.82).

$$
\begin{aligned}
T_{e2} &= \\
&\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{pmatrix}
L_{q11}i_{q1} + L_{q1d1}i_{d1} + L_{q101}i_{01} + L_{q1q2}i_{q2} + L_{q1d2}i_{d2} + L_{q102}i_{02} + L_{q1q3}i_{q3} + L_{q1d3}i_{d3} + L_{q103}i_{03} \\
L_{d1q1}i_{q1} + L_{d11}i_{d1} + L_{d101}i_{01} + L_{d1q2}i_{q2} + L_{d1d2}i_{d2} + L_{d102}i_{02} + L_{d1q3}i_{q3} + L_{d1d3}i_{d3} + L_{d103}i_{03} \\
L_{01q1}i_{q1} + L_{01d1}i_{d1} + L_{0101}i_{01} + L_{01q2}i_{q2} + L_{01d2}i_{d2} + L_{0102}i_{02} + L_{01q3}i_{q3} + L_{01d3}i_{d3} + L_{0103}i_{03} \\
L_{q1q2}i_{q1} + L_{q1d2}i_{d1} + L_{q102}i_{01} + L_{q22}i_{q2} + L_{q2d2}i_{d2} + L_{q202}i_{02} + L_{q2q3}i_{q3} + L_{q2d3}i_{d3} + L_{q203}i_{03} \\
L_{d1q2}i_{q1} + L_{d1d2}i_{d1} + L_{d102}i_{01} + L_{d2q2}i_{q2} + L_{d22}i_{d2} + L_{d202}i_{02} + L_{d2q3}i_{q3} + L_{d2d3}i_{d3} + L_{d203}i_{03} \\
L_{01q2}i_{q1} + L_{01d2}i_{d1} + L_{0102}i_{01} + L_{02q2}i_{q2} + L_{02d2}i_{d2} + L_{0202}i_{02} + L_{02q3}i_{q3} + L_{02d3}i_{d3} + L_{0203}i_{03} \\
L_{q1q3}i_{q1} + L_{q1d3}i_{d1} + L_{q103}i_{01} + L_{q2q3}i_{q2} + L_{q2d3}i_{d2} + L_{q203}i_{02} + L_{q3q3}i_{q3} + L_{q3d3}i_{d3} + L_{q303}i_{03} \\
L_{d1q3}i_{q1} + L_{d1d3}i_{d1} + L_{d103}i_{01} + L_{d2q3}i_{q2} + L_{d2d3}i_{d2} + L_{d203}i_{02} + L_{d3q3}i_{q3} + L_{d3d3}i_{d3} + L_{d303}i_{03} \\
L_{01q3}i_{q1} + L_{01d3}i_{d1} + L_{0103}i_{01} + L_{02q3}i_{q2} + L_{02d2}i_{d2} + L_{0203}i_{02} + L_{03q3}i_{q3} + L_{03d3}i_{d3} + L_{0303}i_{03}
\end{pmatrix} \\
&+ \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix}
\end{aligned} \tag{3.82}
$$

After simplifying the equation (3.82), the non-zero terms are:

$$T_{e2} =$$

$$\frac{3}{2}\frac{P}{2}\left[\begin{matrix}\left(\begin{matrix}L_{q1q2}i_{q1} + L_{q1d2}i_{d1} + L_{q102}i_{01} + L_{q22}i_{q2} + L_{q2d2}i_{d2} + \\ L_{q202}i_{02} + L_{q2q3}i_{q3} + L_{q2d3}i_{d3} + L_{q203}i_{03}\end{matrix}\right)i_{d2} - \\ \left(\begin{matrix}L_{d1q2}i_{q1} + L_{d1d2}i_{d1} + L_{d102}i_{01} + L_{d2q2}i_{q2} + L_{d22}i_{d2} + \\ L_{d202}i_{02} + L_{d2q3}i_{q3} + L_{d2d3}i_{d3} + L_{d203}i_{03}\end{matrix}\right)i_{q2} + \\ \left(\begin{matrix}L_{01q2}i_{q1} + L_{01d2}i_{d1} + L_{0102}i_{01} + L_{02q2}i_{q2} + L_{02d2}i_{d2} + \\ L_{0202}i_{02} + L_{02q3}i_{q3} + L_{02d3}i_{d3} + L_{0203}i_{03}\end{matrix}\right)i_{02}\end{matrix}\right] \tag{3.83}$$

$$+\frac{3}{2}\frac{P}{2}\left(\lambda_{pmq2}i_{d2} + \lambda_{pmd2}i_{q2} + \lambda_{pm02}i_{02}\right)$$

Similarly, the co-energy of the machine 3 also can be presented as function of the stator currents and the flux linkages as:

$$W_{co3} = \frac{1}{2}i_{abc3}{}^{t}L_{ss}i_{abc1,2,3} + i_{s1}{}^{t}\lambda_{pm\_3} \tag{3.84}$$

From the torque co-energy equation, the electromagnetic torque can be derived as:

$$T_{e3} = \frac{\partial W_{co3}}{\partial \theta_{rm}} \tag{3.85}$$

The equation (3.85) is equal to the below one.

$$T_{e3} = \frac{1}{2}i_{abc3}{}^{t}\frac{\partial L_{ss}}{\partial \theta_{rm}}i_{abc1,2,3} + i_{abc3}{}^{t}\frac{\partial \lambda_{pm\_3}}{\partial \theta_{rm}} \tag{3.86}$$

Again the mechanical angle of the equation (3.86) needs to be converted to the electrical angle as:

$$\theta_r = \frac{P}{2}\theta_{rm} \tag{3.87}$$

Therefore, the torque equation changes to the equation (8.88).

$$T_{e3} = \frac{P}{2}\frac{1}{2} i_{abc3}{}^{t} \frac{\partial L_{ss}}{\partial \theta_r} i_{abc1,2,3} + \frac{P}{2} i_{s3}{}^{t} \frac{\partial \lambda_{pm\_3}}{\partial \theta_r} \tag{3.88}$$

Substituting the stator currents with their corresponding values in rotor reference frame results in:

$$T_{e3} = \frac{3}{2}\frac{P}{2}(i_{qdo3})^t\, T(\theta_r)\frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} i_{qdo1,2,3} + \frac{3}{2}\frac{P}{2}(i_{qdo3})^t\, T(\theta_r)\frac{\partial \lambda_{pm\_3}}{\partial \theta_r} \tag{3.89}$$

The equation can be rewritten as:

$$T_{e3} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{q3} \\ i_{o3} \end{bmatrix}^t T(\theta_r)\frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{q3} \\ i_{o3} \end{bmatrix}^t T(\theta_r)\frac{\partial \lambda_{pm-3}}{\partial \theta_r} \tag{3.90}$$

Equation (3.90) is equal to equation (3.91).

$$T_{e3} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{q3} \\ i_{o3} \end{bmatrix}^t L_{qdo}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{q3} \\ i_{o3} \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix} \quad (3.91)$$

Replacing the qd inductances matrix into the equation (3.91) results in:

$$T_{e3} =$$

$$\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix}^t \begin{pmatrix} L_{q11} & L_{q1d1} & L_{q101} & L_{q1q2} & L_{q1d2} & L_{q102} & L_{q1q3} & L_{q1d3} & L_{q103} \\ L_{d1q1} & L_{d11} & L_{d101} & L_{d1q2} & L_{d1d2} & L_{d102} & L_{d1q3} & L_{d1d3} & L_{d103} \\ L_{01q1} & L_{01d1} & L_{0101} & L_{01q2} & L_{01d2} & L_{0102} & L_{01q3} & L_{01d3} & L_{0103} \\ L_{q1q2} & L_{q1d2} & L_{q102} & L_{q22} & L_{q2d2} & L_{q202} & L_{q2q3} & L_{q2d3} & L_{q203} \\ L_{d1q2} & L_{d1d2} & L_{d102} & L_{d2q2} & L_{d22} & L_{d202} & L_{d2q3} & L_{d2d3} & L_{d203} \\ L_{01q2} & L_{01d2} & L_{0102} & L_{02q2} & L_{02d2} & L_{0202} & L_{02q3} & L_{02d3} & L_{0203} \\ L_{q1q3} & L_{q1d3} & L_{q103} & L_{q2q3} & L_{q2d3} & L_{q203} & L_{q33} & L_{q3d3} & L_{q303} \\ L_{d1q3} & L_{d1d3} & L_{d103} & L_{d2q3} & L_{d2d3} & L_{d203} & L_{d3q3} & L_{d33} & L_{d303} \\ L_{01q3} & L_{01d3} & L_{0103} & L_{02q3} & L_{02d3} & L_{0203} & L_{03q3} & L_{03d3} & L_{0303} \end{pmatrix} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \tag{3.92}$$

$$\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix}$$

The equation (3.92) is equal to:

$$
T_{e3} =
$$

$$
\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix}^t \begin{pmatrix}
L_{q11}i_{q1} + L_{q1d1}i_{d1} + L_{q101}i_{01} + L_{q1q2}i_{q2} + L_{q1d2}i_{d2} + L_{q102}i_{02} + L_{q1q3}i_{q3} + L_{q1d3}i_{d3} + L_{q103}i_{03} \\
L_{d1q1}i_{q1} + L_{d11}i_{d1} + L_{d101}i_{01} + L_{d1q2}i_{q2} + L_{d1d2}i_{d2} + L_{d102}i_{02} + L_{d1q3}i_{q3} + L_{d1d3}i_{d3} + L_{d103}i_{03} \\
L_{01q1}i_{q1} + L_{01d1}i_{d1} + L_{0101}i_{01} + L_{01q2}i_{q2} + L_{01d2}i_{d2} + L_{0102}i_{02} + L_{01q3}i_{q3} + L_{01d3}i_{d3} + L_{0103}i_{03} \\
L_{q1q2}i_{q1} + L_{q1d2}i_{d1} + L_{q102}i_{01} + L_{q22}i_{q2} + L_{q2d2}i_{d2} + L_{q202}i_{02} + L_{q2q3}i_{q3} + L_{q2d3}i_{d3} + L_{q203}i_{03} \\
L_{d1q2}i_{q1} + L_{d1d2}i_{d1} + L_{d102}i_{01} + L_{d2q2}i_{q2} + L_{d22}i_{d2} + L_{d202}i_{02} + L_{d2q3}i_{q3} + L_{d2d3}i_{d3} + L_{d203}i_{03} \\
L_{01q2}i_{q1} + L_{01d2}i_{d1} + L_{0102}i_{01} + L_{02q2}i_{q2} + L_{02d2}i_{d2} + L_{0202}i_{02} + L_{02q3}i_{q3} + L_{02d3}i_{d3} + L_{0203}i_{03} \\
L_{q1q3}i_{q1} + L_{q1d3}i_{d1} + L_{q103}i_{01} + L_{q2q3}i_{q2} + L_{q2d3}i_{d2} + L_{q203}i_{02} + L_{q3q3}i_{q3} + L_{q3d3}i_{d3} + L_{q303}i_{03} \\
L_{d1q3}i_{q1} + L_{d1d3}i_{d1} + L_{d103}i_{01} + L_{d2q3}i_{q2} + L_{d2d3}i_{d2} + L_{d203}i_{02} + L_{d3q3}i_{q3} + L_{d3d3}i_{d3} + L_{d303}i_{03} \\
L_{01q3}i_{q1} + L_{01d3}i_{d1} + L_{0103}i_{01} + L_{02q3}i_{q2} + L_{02d2}i_{d2} + L_{0203}i_{02} + L_{03q3}i_{q3} + L_{03d3}i_{d3} + L_{0303}i_{03}
\end{pmatrix} \tag{3.93}
$$

$$
+\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix}
$$

After simplifying the non-zero terms of the equation (3.93) are:

$$
T_{e3} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} \begin{pmatrix} L_{q1q3}i_{q1} + L_{q1d3}i_{d1} + L_{q103}i_{01} + L_{q2q3}i_{q2} + L_{q2d3}i_{d2} \\ + L_{q203}i_{02} + L_{q3q3}i_{q3} + L_{q3d3}i_{d3} + L_{q303}i_{03} \end{pmatrix} i_{d3} - \\ \begin{pmatrix} L_{d1q3}i_{q1} + L_{d1d3}i_{d1} + L_{d103}i_{01} + L_{d2q3}i_{q2} + L_{d2d3}i_{d2} \\ + L_{d203}i_{02} + L_{d3q3}i_{q3} + L_{d3d3}i_{d3} + L_{d303}i_{03} \end{pmatrix} i_{q3} + \\ \begin{pmatrix} L_{01q3}i_{q1} + L_{01d3}i_{d1} + L_{0103}i_{01} + L_{02q3}i_{q2} + L_{02d2}i_{d2} \\ + L_{0203}i_{02} + L_{03q3}i_{q3} + L_{03d3}i_{d3} + L_{0303}i_{03} \end{pmatrix} i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2}\left(\lambda_{pmq3}i_{d3} + \lambda_{pmd2}i_{q3} + \lambda_{pm02}i_{03}\right) \quad (3.94)
$$

The dynamic equation of the rotor speed can also be derived using the electromagnetic and load torque. In equation (3.95) *'P'* is the number of the pole of the machine, 'ω_r' is the rotor speed, 'B' is the friction coefficient and 'T_L' is the mechanical load torque applied to the machine.

$$
T_e = T_{e1} + T_{e2} + T_{e3} = J\left(\frac{2}{P}\right)p\omega_r + T_L + B\omega_r \quad (3.95)
$$

3.7.3 Generating the Parameters of the Symmetrical Triple-Star Nine Phase IPM

After generation of the model equations, the parameters of the machine need to be generated. The machine parameters are the different inductances that are determined from the turn and winding functions of the machine and also the flux linkages of the permanent magnets. The clock diagram of the machine is shown in the Figure 3.70 (a). As it can be seen each machine has four poles with full pitch, double layer and concentrated windings.

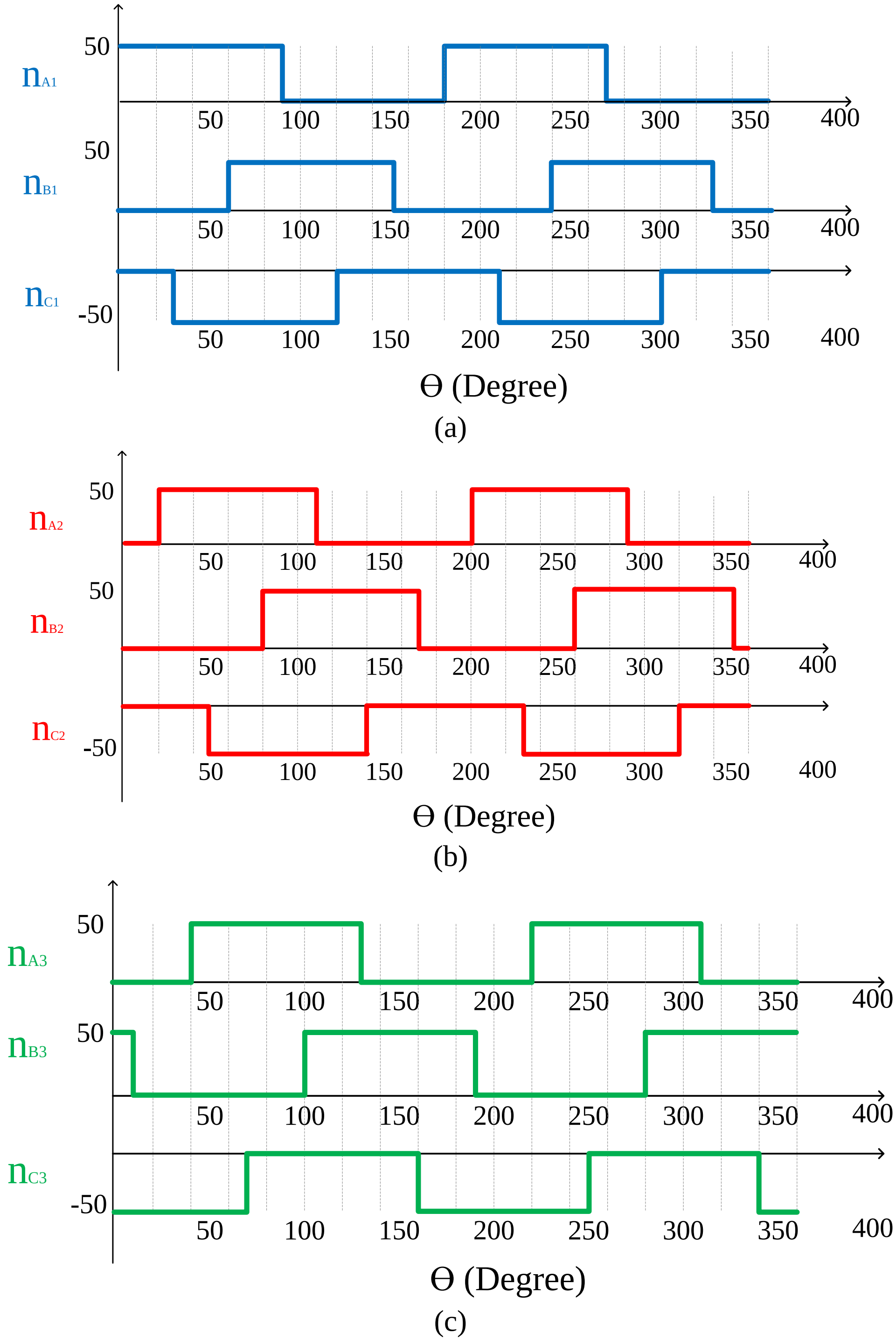


Figure 3.71: The turn functions of the, (a) Machine ‘1’ phases, (b) Machine ‘2’ phases, (c) Machine ‘3’ phases.

The machine has 36 slots and each slot covers 40 degrees of the stator circumferential and also each machine is shifted by 40 degrees from the adjacent machine. In this diagram the machine 1 is shown in blue, machine 2 is shown in red and machine 3 is shown in green color. The turn function of each machine can be generated from the clock diagram. Figures 3.71 shows the turn function for each of the machines. The rotor of the machine is the same as the single star machine, shown in the Figure 3.5. It can be seen that the rotor has four bars of the permanent magnet materials buried inside. Again considering the effect of the permanent magnet bars on the effective airgap, the airgap function can be generated as Figure 3.6.

Using the above figures, the equation (3.96) the winding functions of the phase stator phases are generated as a function of rotor and circumferential angle of the stator. Figure 3.72 shows the winding function of the phase 'a' of the machine 1 [83].

$$N_w(\theta) = n_w(\theta) - \frac{\int_0^{2\pi} \frac{n_w(\theta)}{g(\theta,\theta_r)} d\theta}{\int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} d\theta} \tag{3.96}$$

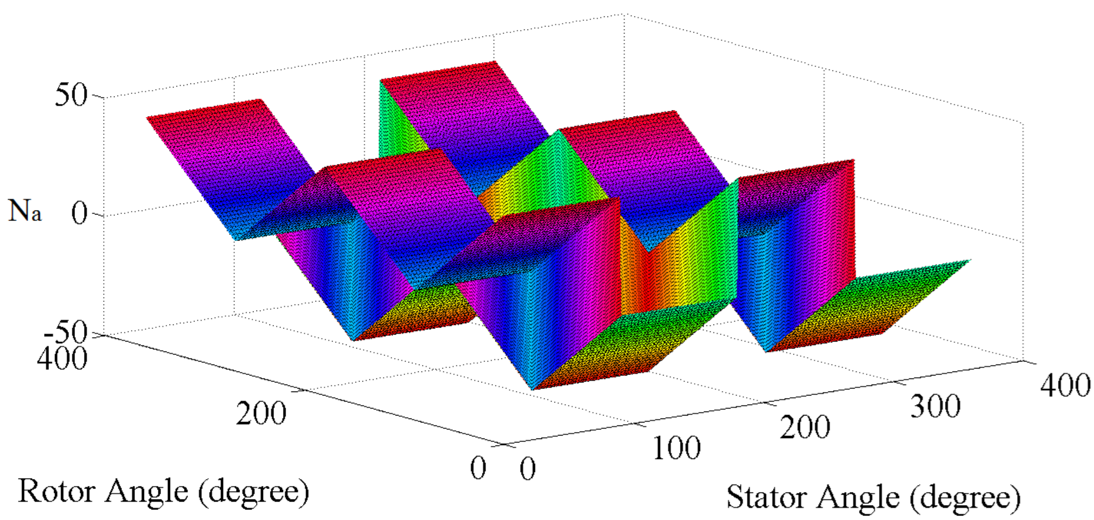


Figure 3.72: The winding function of the phase 'a' of the machine 1.

The winding functions of the rest of the phases have the same form except they are shifted by 40 degree of the stator angle.

Using the generated winding functions and equation (3.97) the self and mutual inductances of the machine phases can be calculated. Figures 3.73 to 3.78 show the mutual and self-inductances of the machine corresponding to each stator phases. In equation (3.97) 'r' is the radius of the rotor, 'l' is the machine length, 'μ_0' is the permeability of the machine iron, 'n_j' is the turn function of the phase 'j', 'N_k' is the winding function of the phase 'k' and '$g(\theta,\theta_r)$' is the air gap function of the machine.

$$L_{jk} = \mu_o r l \int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} n_j(\theta) N_k(\theta) d\theta \qquad (3.97)$$

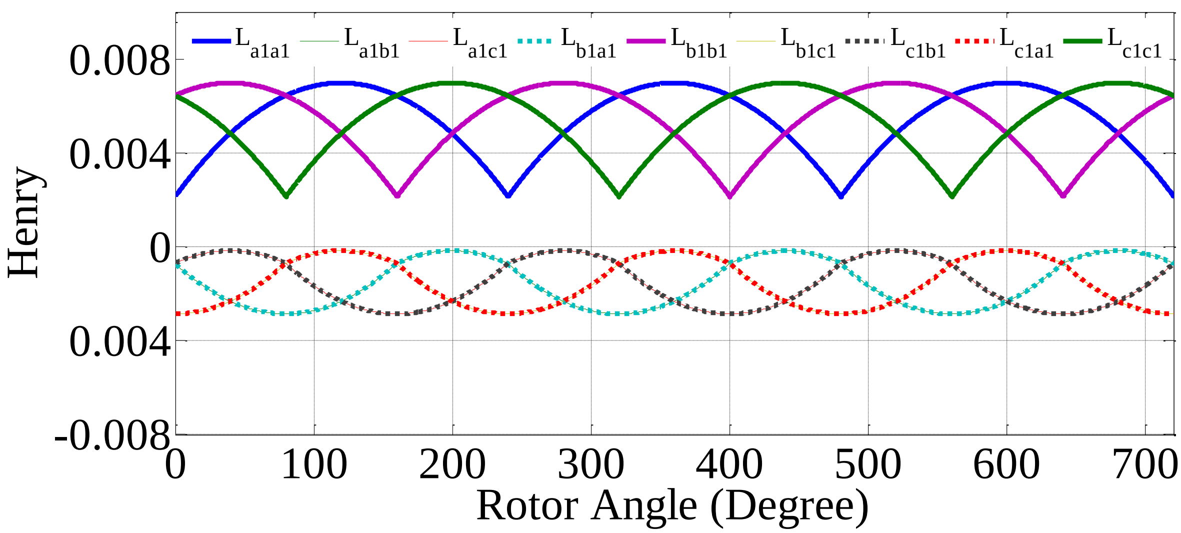


Figure 3.73: The self and mutual inductances corresponding to phases of machine 1.

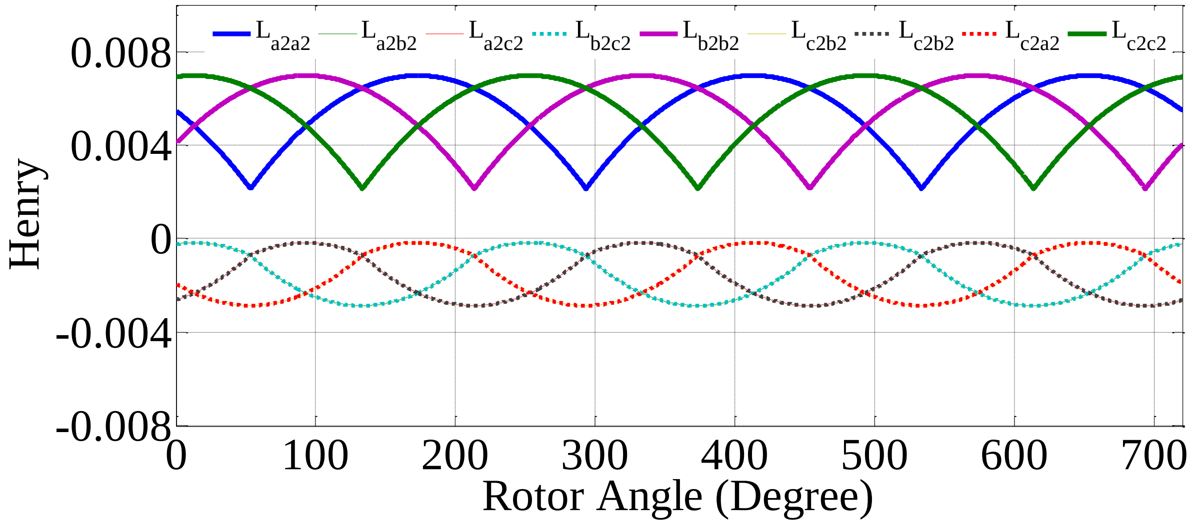


Figure 3.74: The self and mutual inductances corresponding to phases of machine 2.

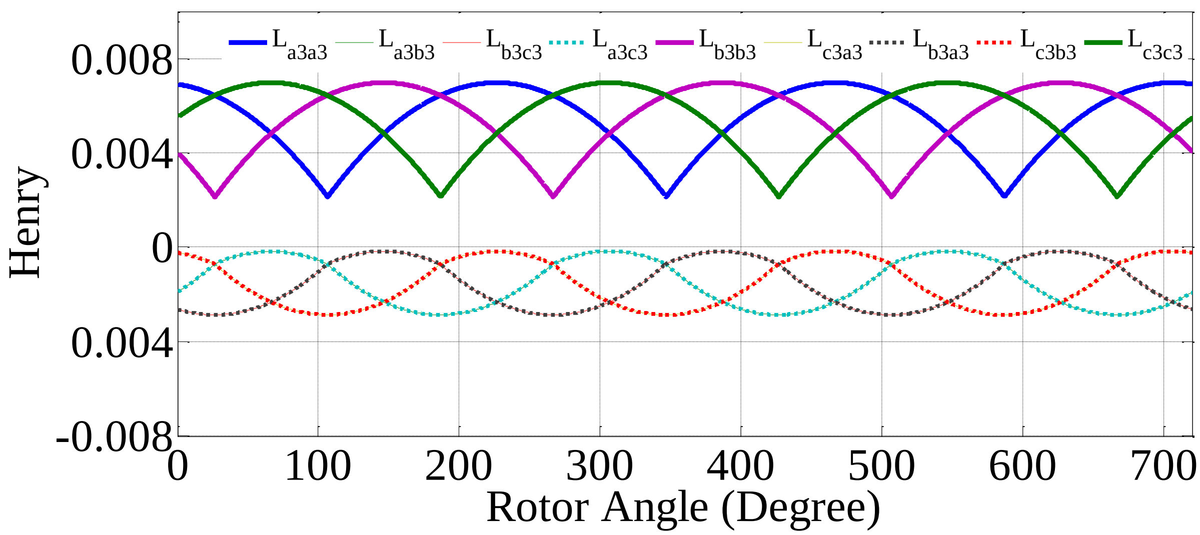


Figure 3.75: The self and mutual inductances corresponding to phases of machine 3.

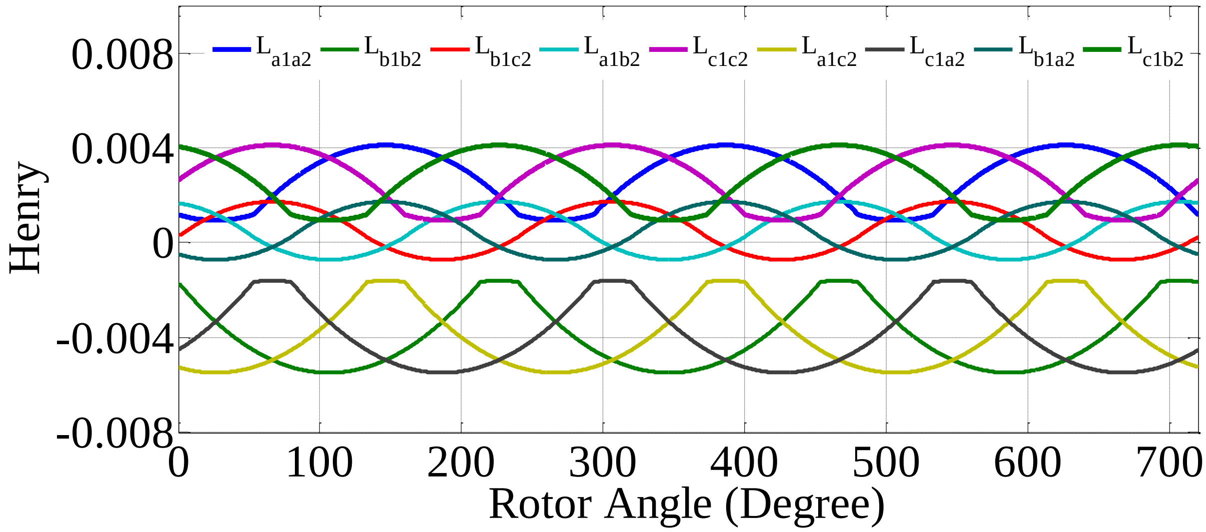


Figure 3.76: The mutual inductances between machines 1 and 2.

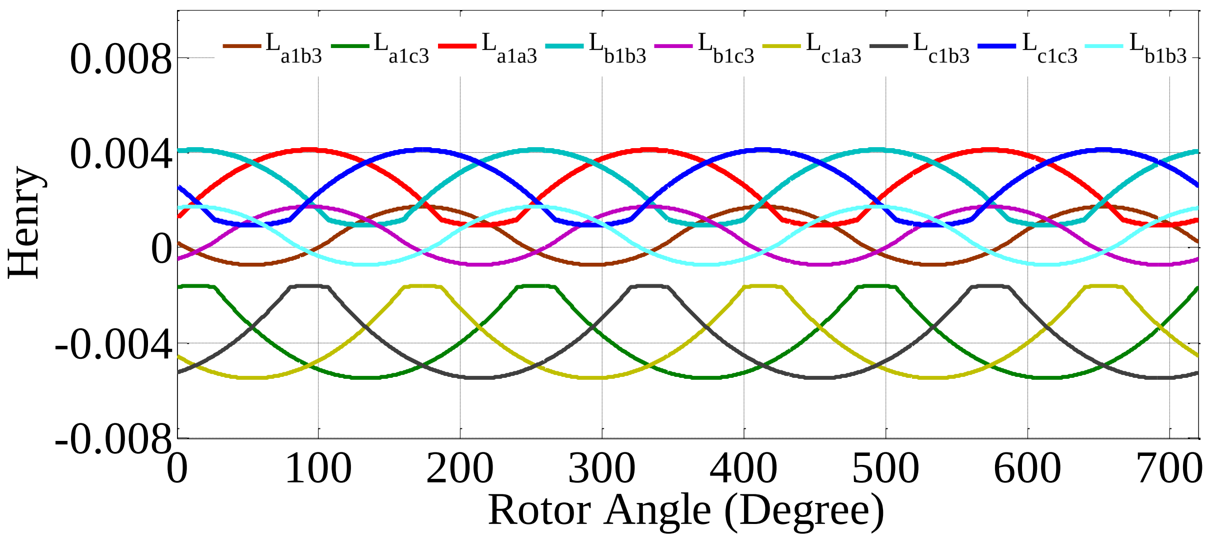


Figure 3.77: The mutual inductances between machines 1 and 3.

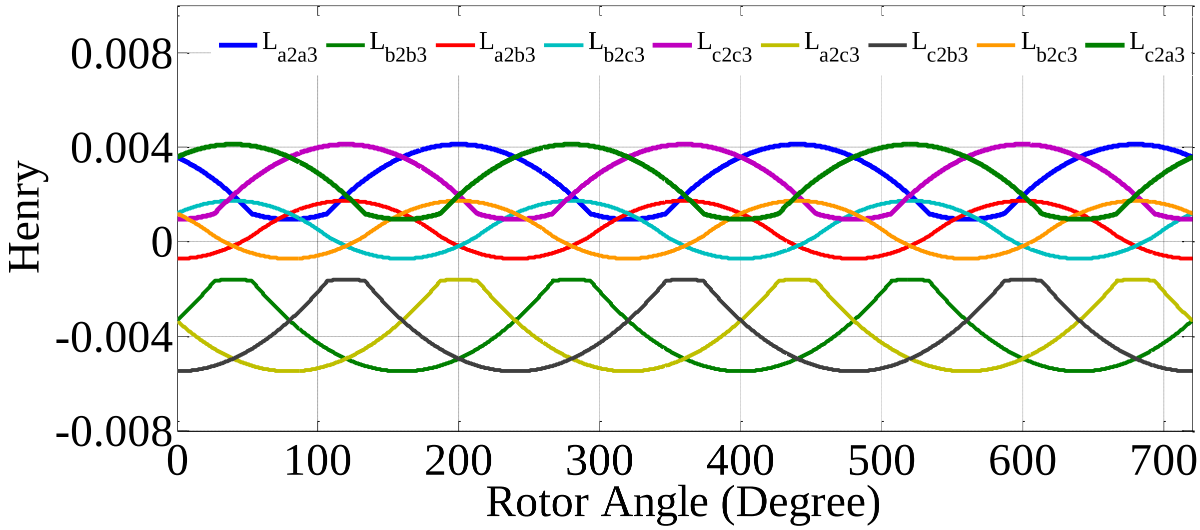


Figure 3.78: The mutual inductances between machines 2 and 3.

Stator self-inductances are maximum when the rotor q-axis is aligned with the phase, and mutual inductances are maximum when the rotor q-axis is in the midway between two phases. Now the inductances can be arranged in the matrix of equation (3.98) and using the equation (3.56), they can be transformed to the rotor reference frame to obtain the q and d inductances of each machine.

$$L_{ss} = \begin{pmatrix} L_{ls}+L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} & L_{a1a3} & L_{a1b3} & L_{a1c3} \\ L_{b1a1} & L_{ls}+L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} & L_{b1a3} & L_{b1b3} & L_{b1c3} \\ L_{c1a1} & L_{c1b1} & L_{ls}+L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} & L_{c1a3} & L_{c1b3} & L_{c1c3} \\ L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{ls}+L_{a2a2} & L_{a2b2} & L_{a2c2} & L_{a2a3} & L_{a2b3} & L_{a2c3} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{ls}+L_{b2b2} & L_{b2c2} & L_{b2a3} & L_{b2b3} & L_{b2c3} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{ls}+L_{c2c2} & L_{c2a3} & L_{c2b3} & L_{c2c3} \\ L_{a3a1} & L_{a3b1} & L_{a3c1} & L_{a3a2} & L_{a3b2} & L_{a3c2} & L_{ls}+L_{a3a3} & L_{a3b3} & L_{a3c3} \\ L_{b3a1} & L_{b3b1} & L_{b3c1} & L_{b3a2} & L_{b3b2} & L_{b3c2} & L_{b3a3} & L_{ls}+L_{b3b3} & L_{b3c3} \\ L_{c3a1} & L_{c3b1} & L_{c3c1} & L_{c3a2} & L_{c3b2} & L_{c3c2} & L_{c3a3} & L_{c3b3} & L_{ls}+L_{c3c3} \end{pmatrix} \quad (3.98)$$

The generated inductances in rotor reference frame are shown in the Figures 3.79 to 3.84.

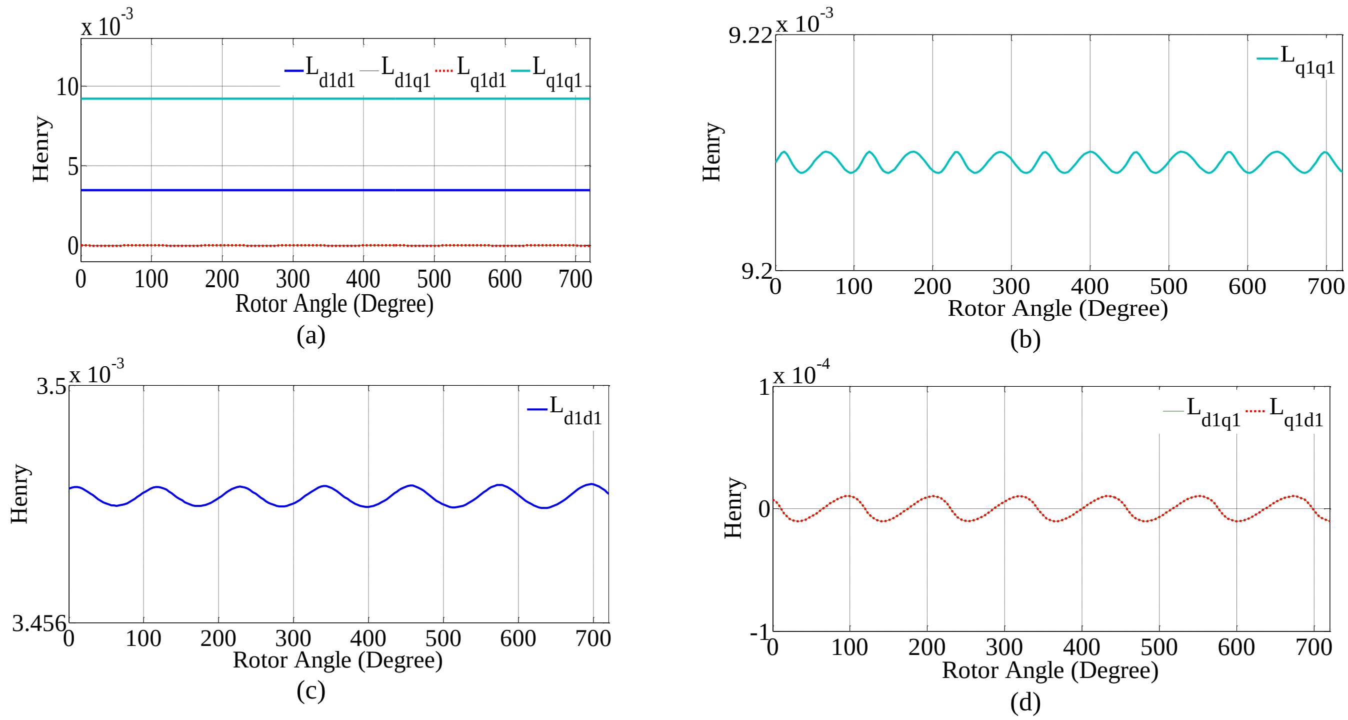


Figure 3.79: (a) The inductances of the machine 1 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

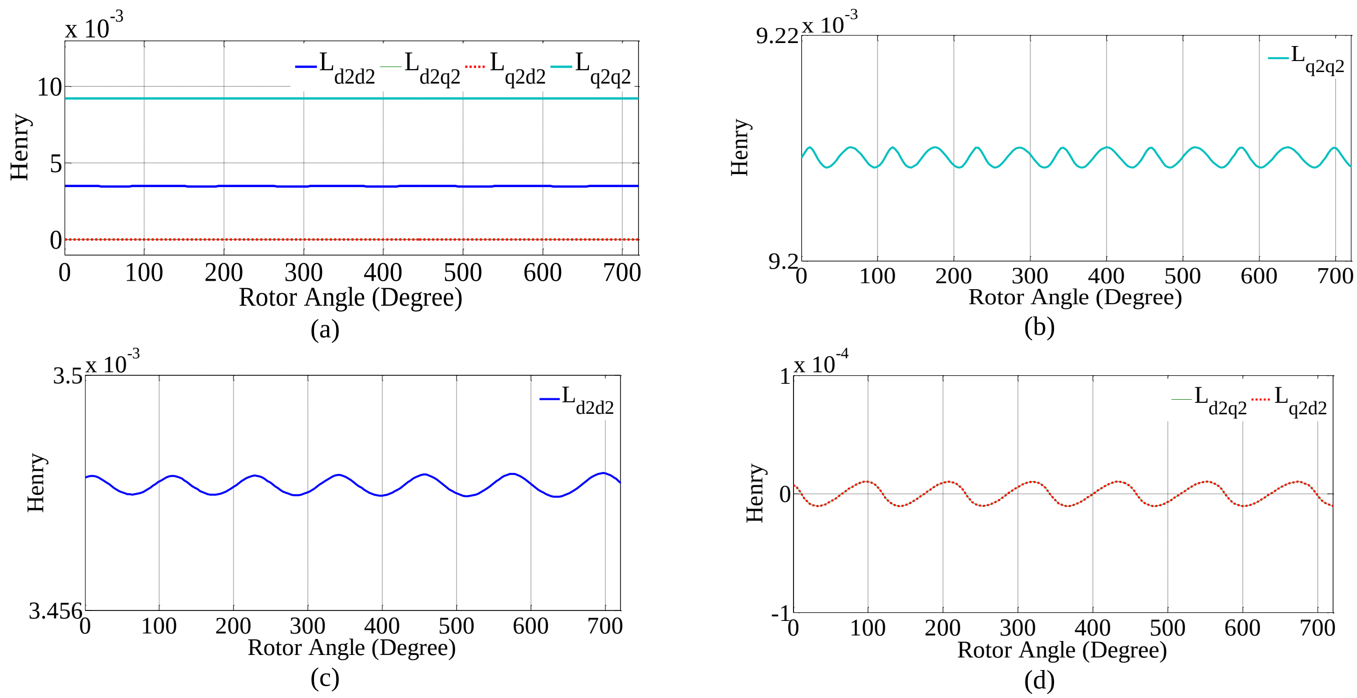


Figure 3.80: (a) The inductances of the machine 2 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

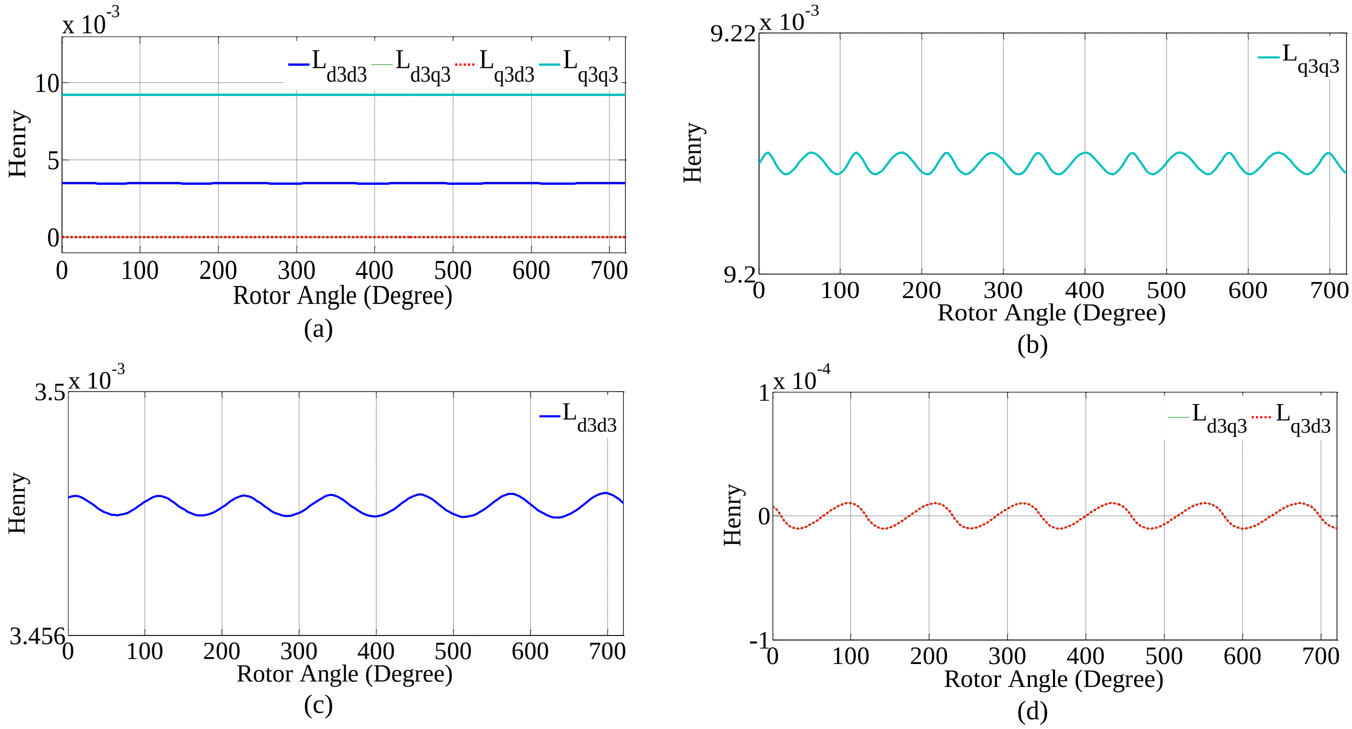


Figure 3.81: (a) The inductances of the machine 3 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

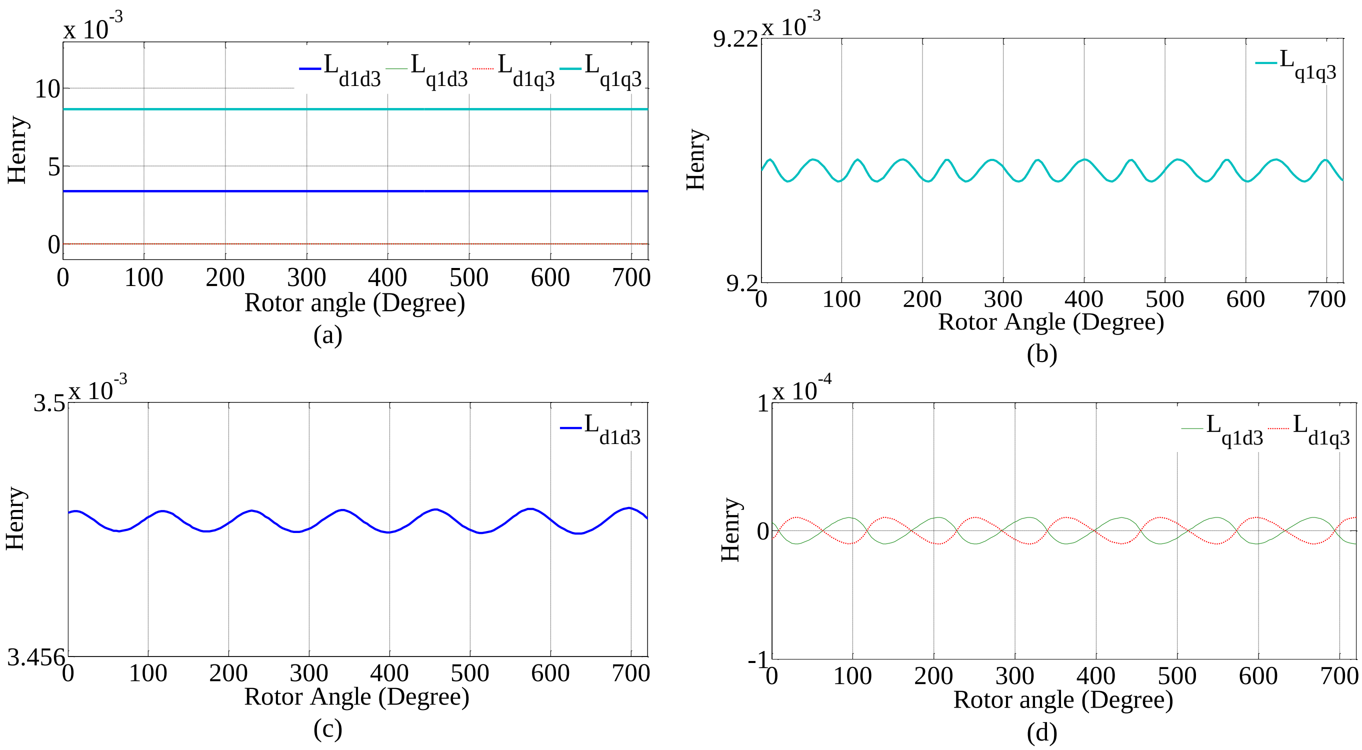


Figure 3.82: (a) The mutual inductances between the machines 1 and 3 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

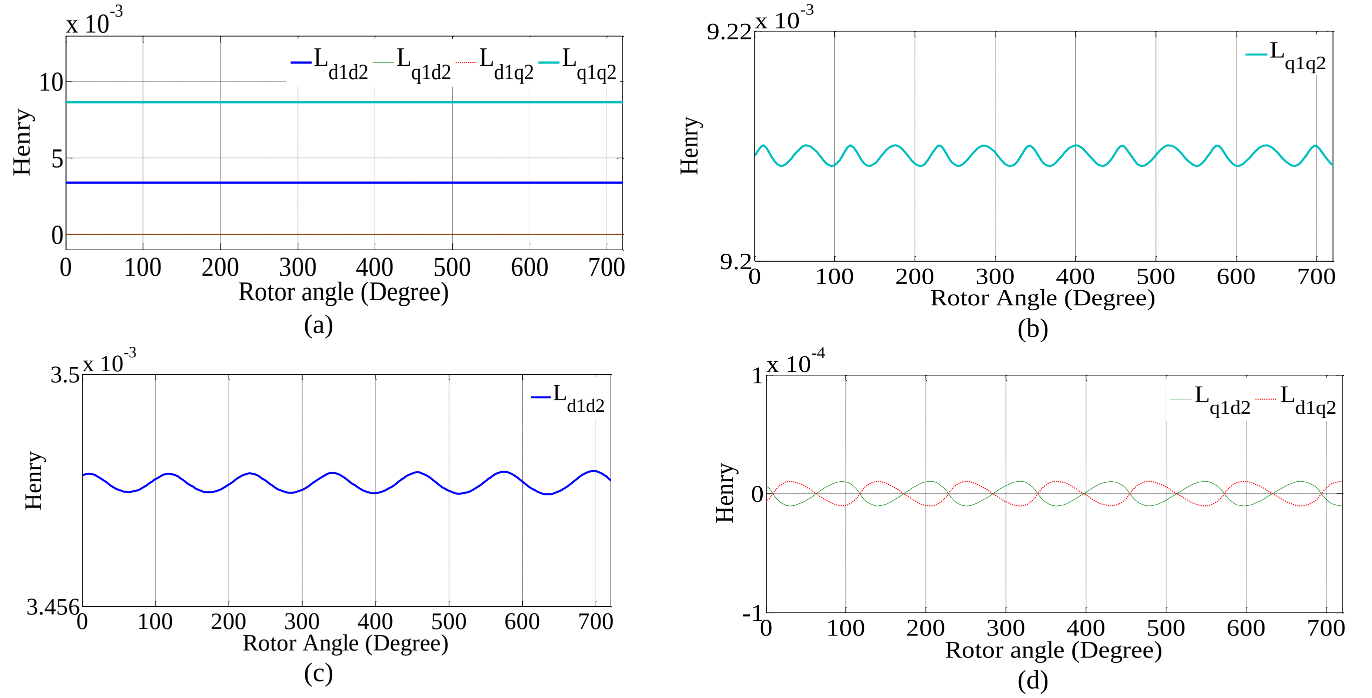


Figure 3.83: (a) The mutual inductances between the machines 1 and 2 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

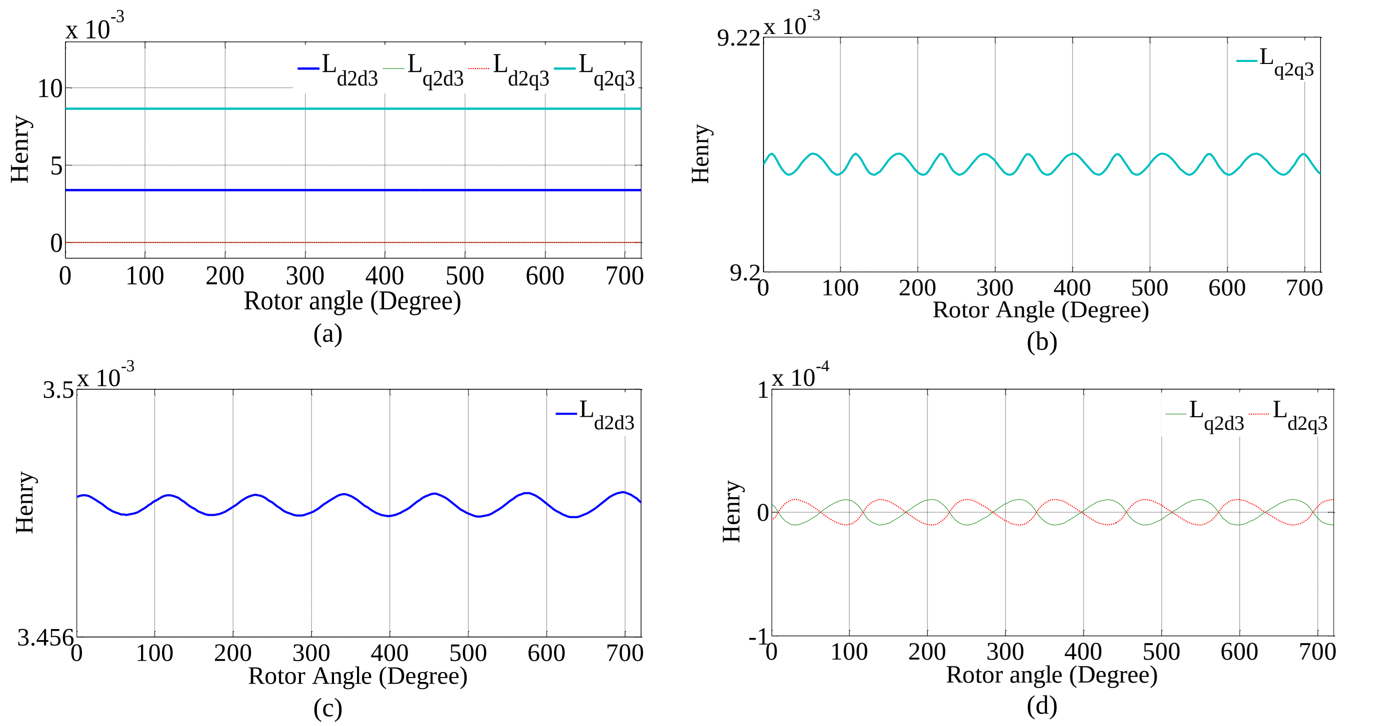


Figure 3.84: (a) The mutual inductances between the machines 2 and 3 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

The equation (3.36) and Figure 3.30, are repeated here to show the flux linkage due to the rotor permanent magnets seen from the phase ‘a’, the flux linkage seen in the rest of the phases have the same shape with 40 degree shift from each other.

$$B_r(\theta_r)=\begin{cases} B_{\max} & -\dfrac{90^o-\alpha_p}{2}<|\theta_r|<\dfrac{90-\alpha_p}{2}\ ,\ 135^o-\alpha_p<|\theta_r|<180^o \\ B_{\max}-\dfrac{2B_{\max}}{\alpha_p}\theta_r & \dfrac{90^o-\alpha_p}{2}<|\theta_r|<90^o \\ -B_{\max} & 90^o<|\theta_r|<135^o-\dfrac{3\alpha_p}{2} \\ -B_{\max}+\dfrac{2B_{\max}}{\alpha_p}\theta_r & 135^o-\dfrac{3\alpha_p}{2}<|\theta_r|<135^o-\alpha_p \end{cases} \tag{3.99}$$

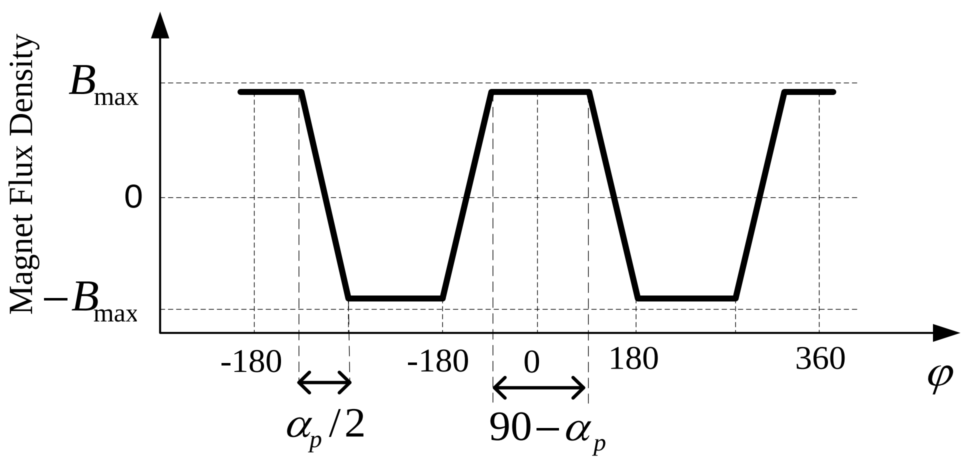


Figure 3.85: Plot of flux density with spatial angle.

Now according to equation (3.55) the flux linkage due to the permanent magnets on each phase of the stator can be transformed to the rotor reference frame to obtain the *q* and *d* axis flux linkages for each of the machines. Figures 3.86 to 3.91 show the flux linkage components.

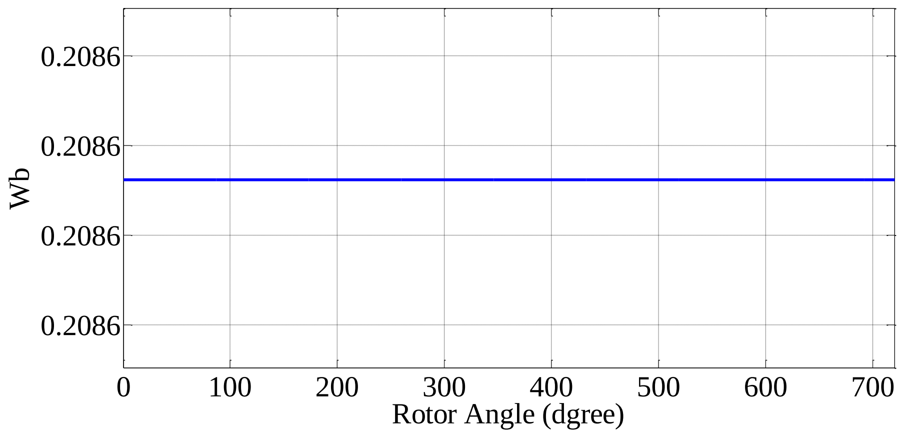


Figure 3.86: The d axis flux linkage due to the rotor permanent magnets of machine 1.

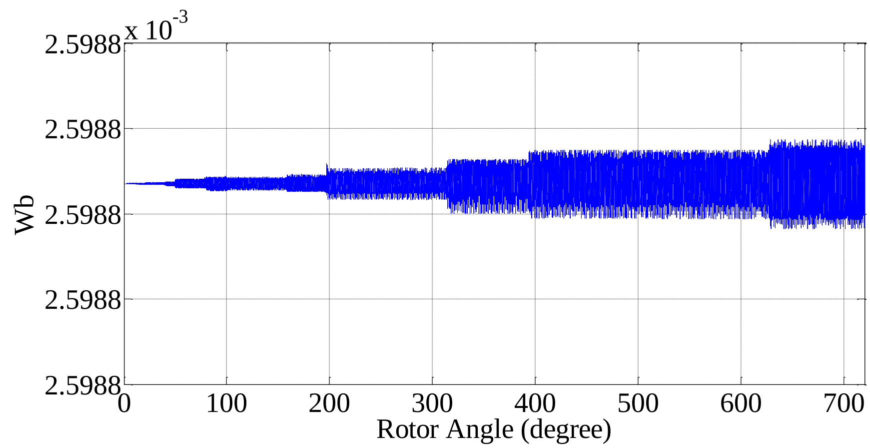


Figure 3.87: The q axis flux linkage due to the rotor permanent magnets of machine 1.

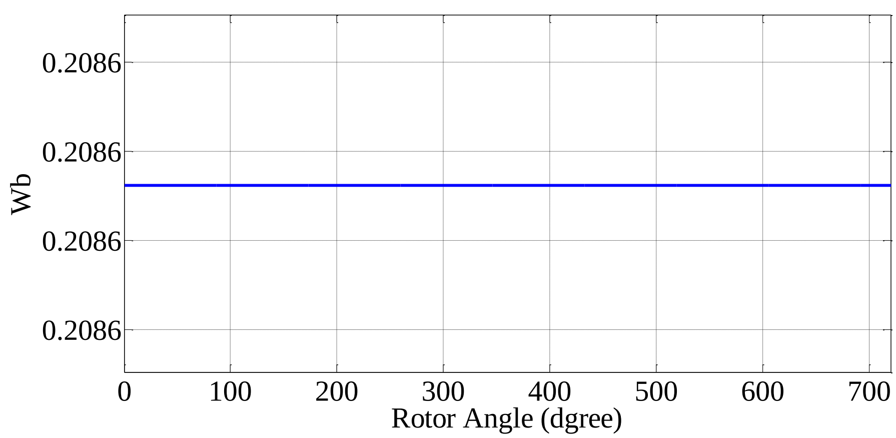


Figure 3.88: The d axis flux linkage due to the rotor permanent magnets of machine 2.

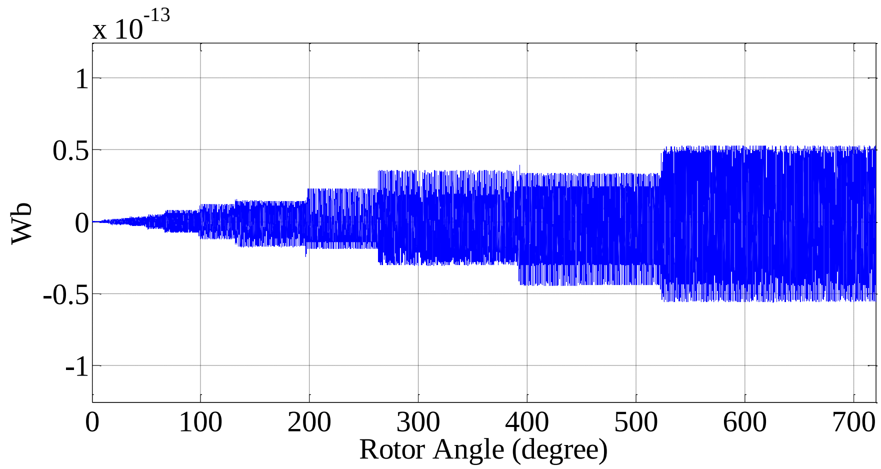


Figure 3.89: The q axis flux linkage due to the rotor permanent magnets of machine 2.

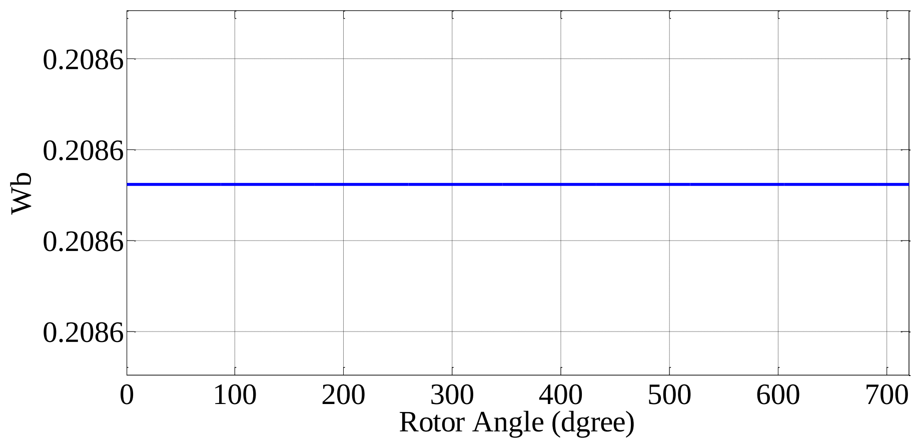


Figure 3.90: The d axis flux linkage due to the rotor permanent magnets of machine 3.

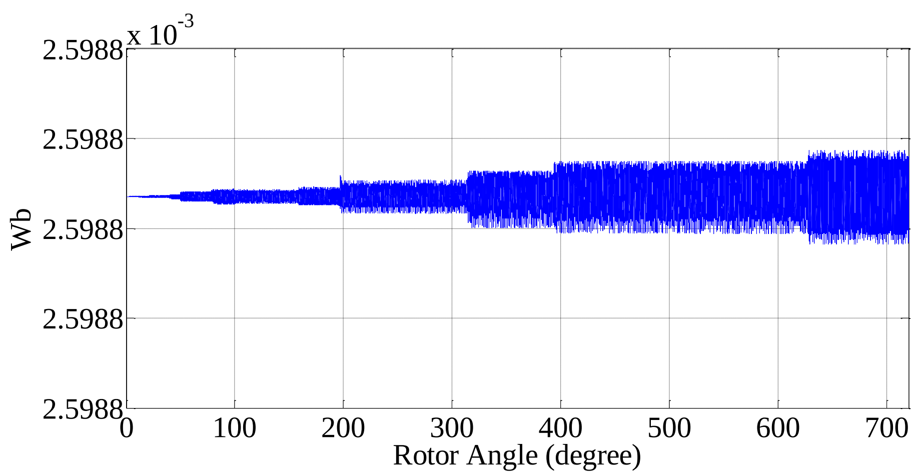


Figure 3.91: The q axis flux linkage due to the rotor permanent magnets of machine 3.

3.7.4 Simulation of the Coupled Model of the Symmetrical Triple Star Nine-Phase IPM

The generated parameters are substituted in the machine model and model is simulated using MATLAB/Simulink. Three sets of 60 (Hz) 110 (Volts) three-phase voltages (as shown in Figure 3.92) are applied to the model while the initial rotor speed is 377 rad/sec. The machine faces some initial transients and after that the rotor speed goes to the synchronous at steady state, a mechanical load torque equal to 5 N.m is applied to the machine. The following figures show the simulation results.

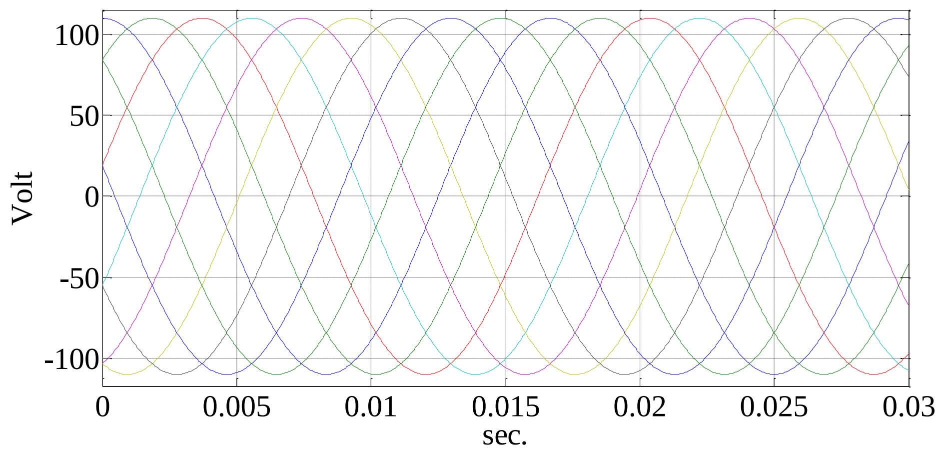


Figure 3.92: The phase voltages.

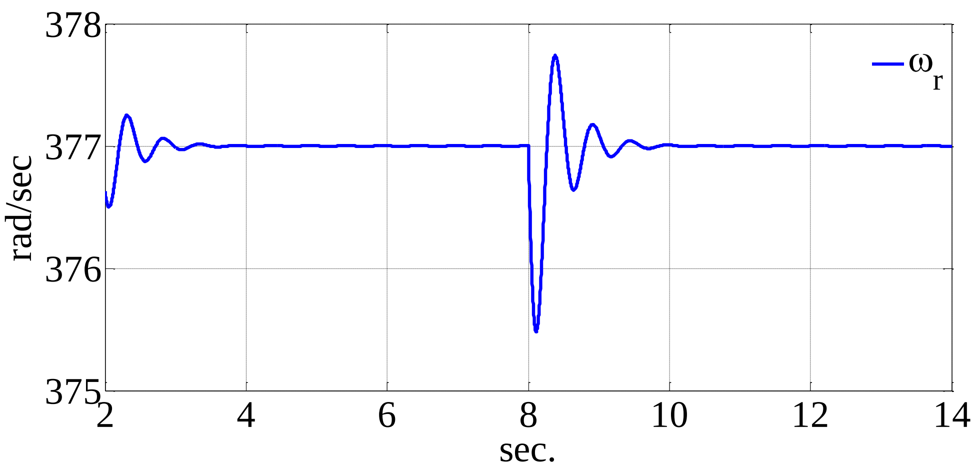


Figure 3.93: The rotor speed.

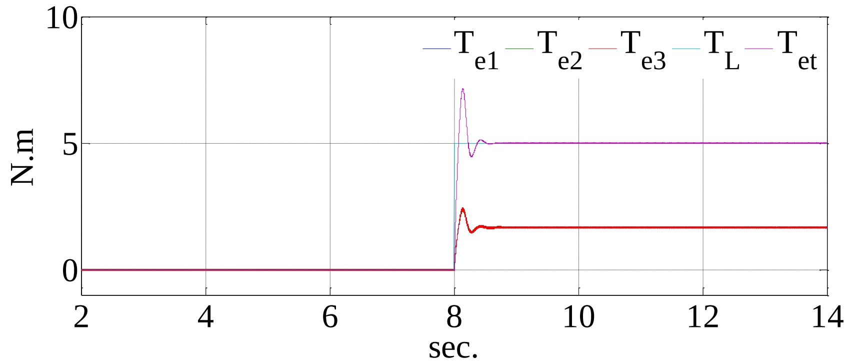


Figure 3.94: The electromagnetic torque generated by machines.

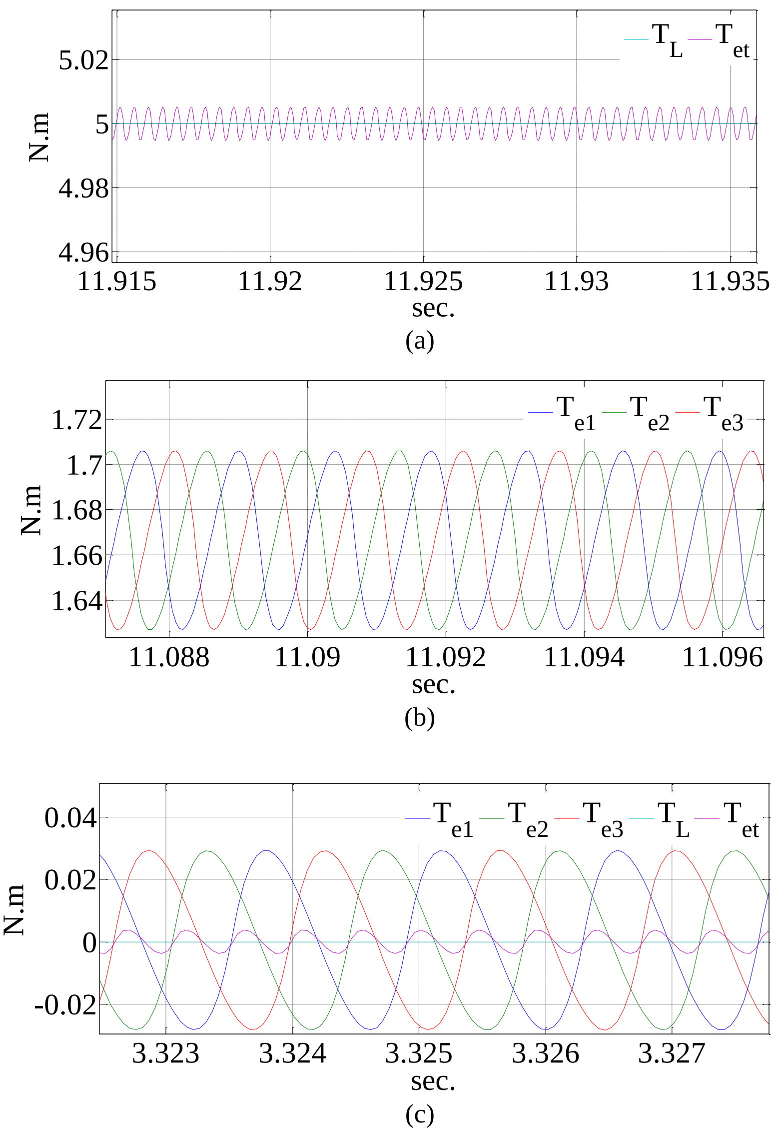


Figure 3.95: The zoomed view of electromagnetic torques generated by machines (a) Total torque, (b) Individual torques of the machines after applying load, (c) Individual torques of the machines before applying load.

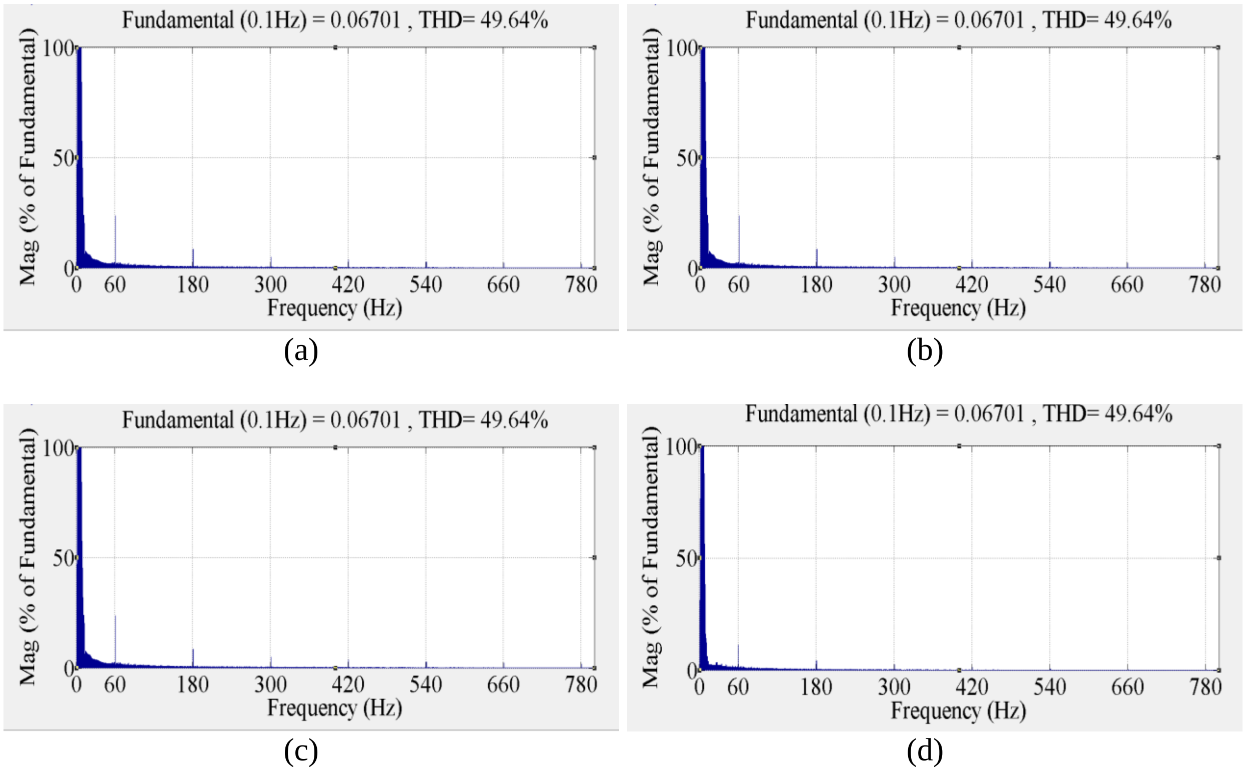


Figure 3.96: The spectrum of the electromagnetic torque for, (a) Machine ‘1’, (b) Machine ‘2’, (c) Machine ‘3’, (d) Total.

Figure 3.93 shows the rotor speed. It can be seen that after initial transients the rotor speed goes to the synchronous speed which is equal to the source frequency. Also when the load torque is applied to the machine due to the changing in the torque angle of the machine the speed shows transients before going back to the steady state. Figure 3.94 shows the electromagnetic torque of each machine and the sum of them. As it can be seen when the transients are over the average electromagnetic torque goes to zero. After applying the load, the machine starts generating electromagnetic torque to keep the synchronous speed. The zoomed view of the electromagnetic torques of each machine and total are shown in Figure 3.95. Also the spectrum of the electromagnetic torque for the total torque and each machine are also shown in the Figures 3.96. The spectrum of the airgap flux linkage of each machine is shown in the Figures 3.97. the harmonics of the flux linkage can be seen beside the main harmonic of the flux linkage. The main harmonic has the frequency of 60 hertz which is equatl to the source frequency.

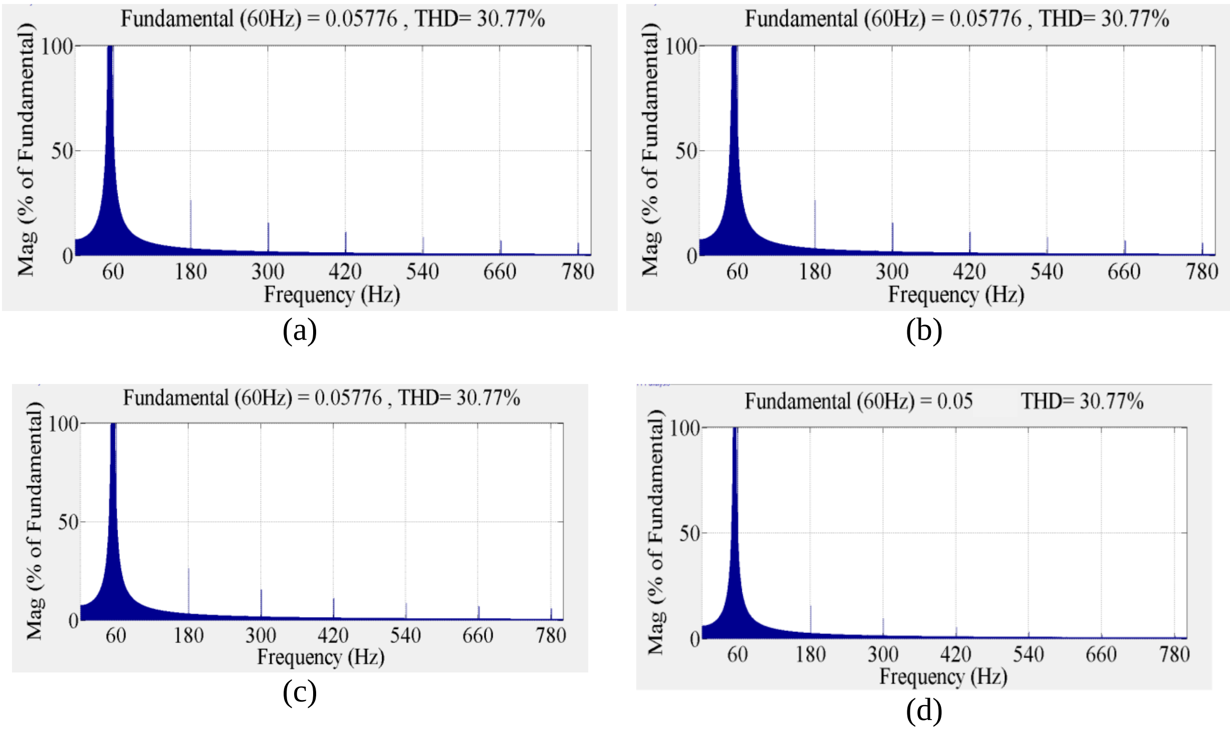
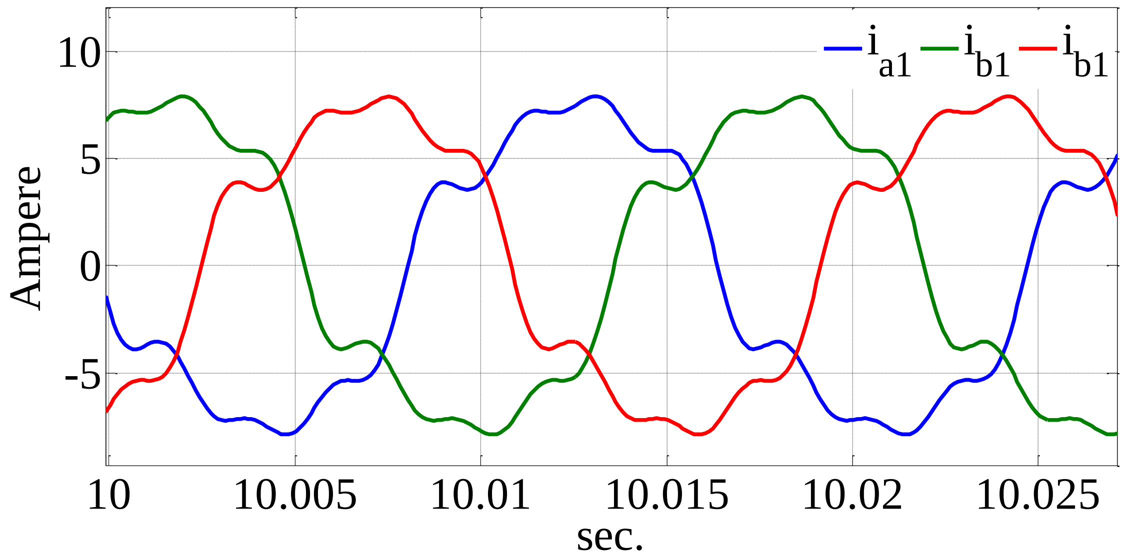
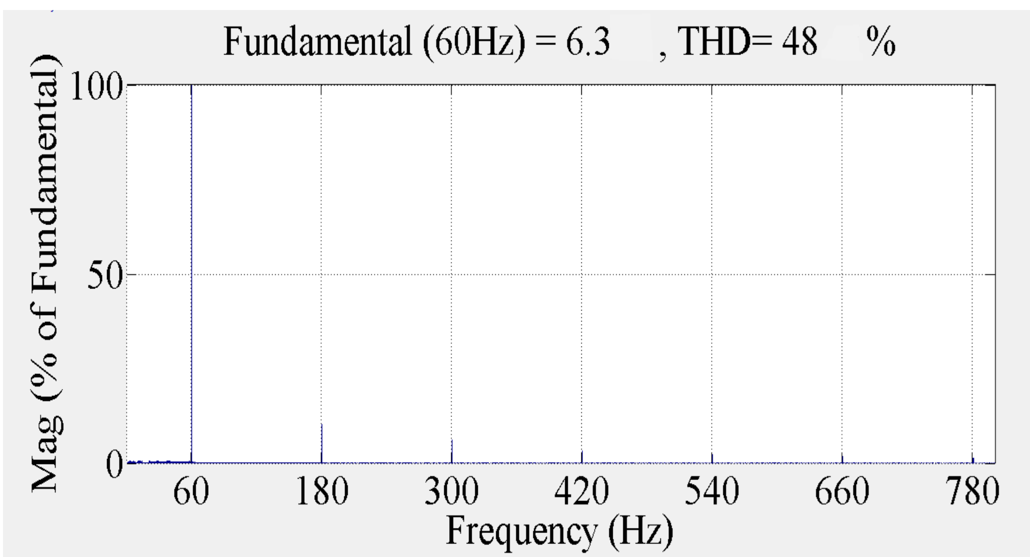


Figure 3.97: The spectrum of the airgap flux linkage for, (a) Machine ‘1’, (b) Machine ‘2’, (c) Machine ‘3’, (d) Total.

The stator currents in natural quantities are shown in the Figures 3.98 to 3.100. The spectrum of the phase ‘a’ of each machine is also shown in these figures. The machine is also tested using the 9 phase setup in the lab under the same conditions for the voltages and the load. Figure 3.101 shows the currents of each machine at the steady state along with the spectrum of the current of the phase ‘a’ of the same machine.

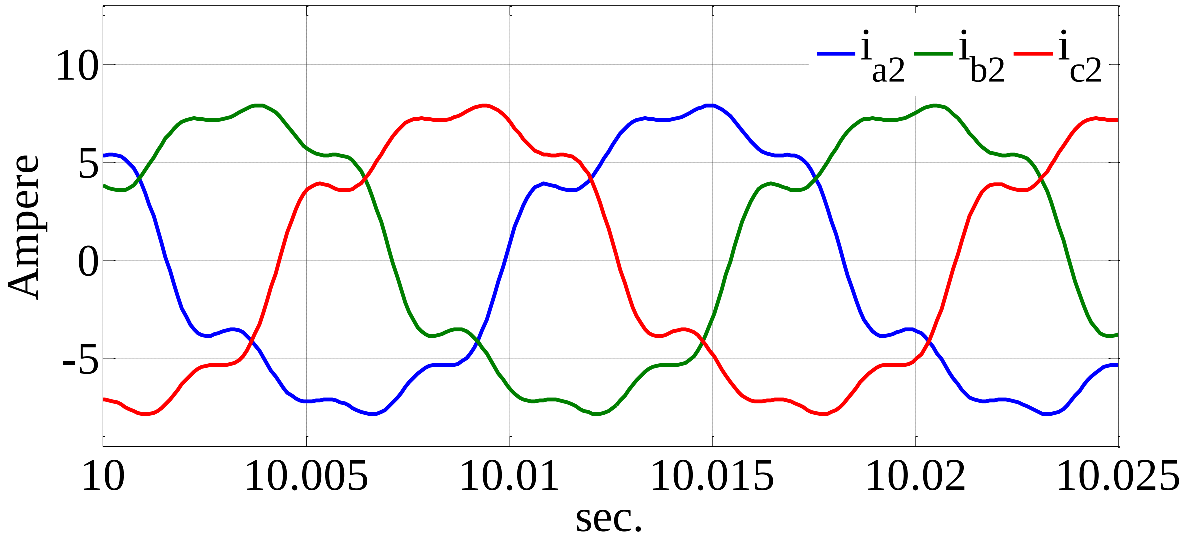


(a)

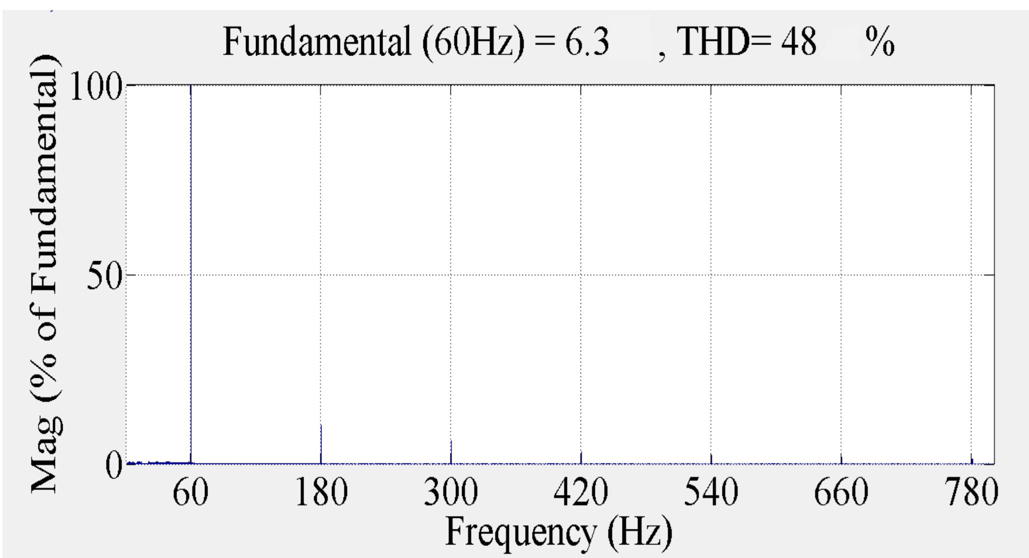


(b)

Figure 3.98: (a) The stator currents of machine 1 at steady state, (b) The spectrum of the phase 'a' current of machine '1'.

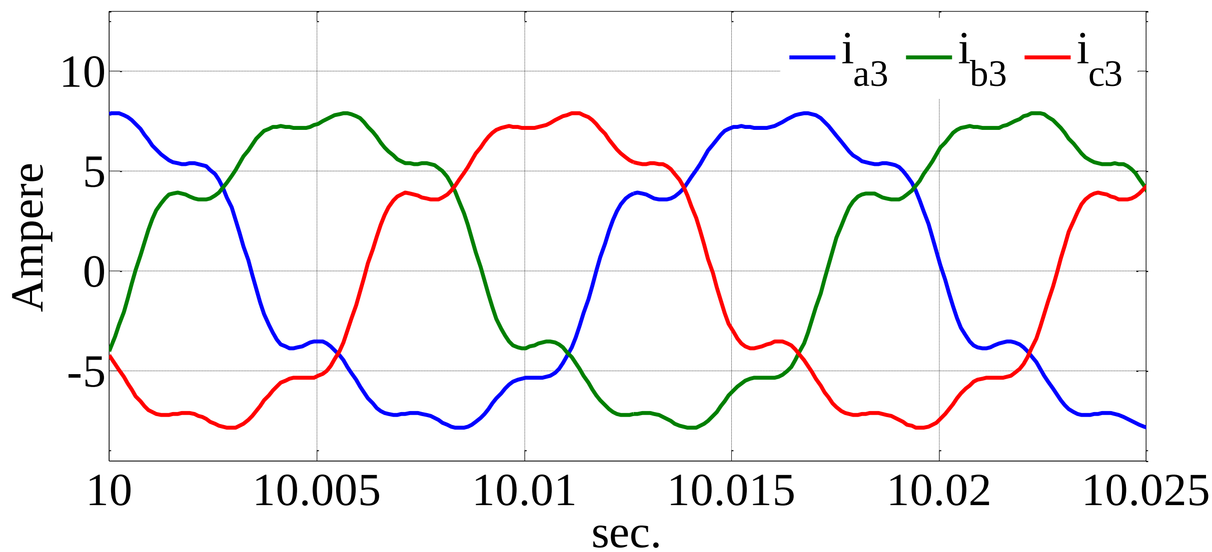


(a)

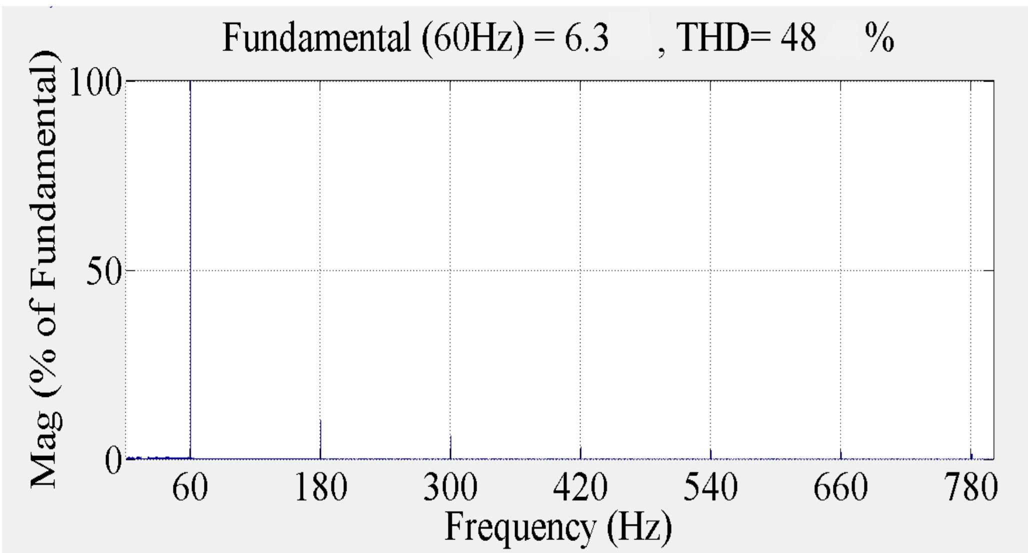


(b)

Figure 3.99: (a) The stator currents of machine 2 at steady state, (b) The spectrum of the phase ‘a’ current of machine ‘2’.



(a)



(b)

Figure 3.100: (a) The stator currents of machine 3 at steady state, (b) The spectrum of the phase 'a' current of machine '3'.

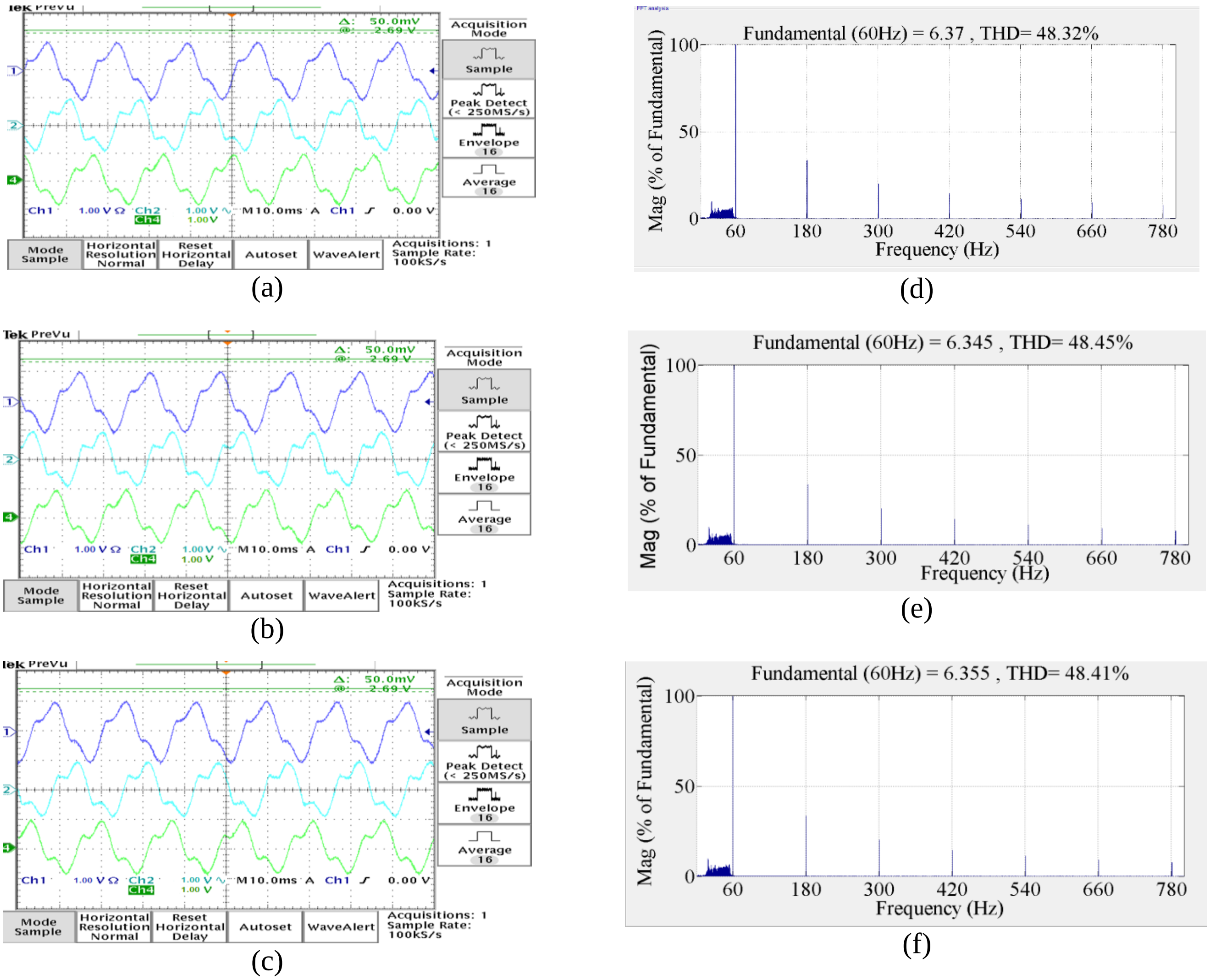


Figure 3.101: The experimental results of the stator currents of machines at steady state, (a) Machine 1, (b) Machine 2, (c) Machine 3 (5 A/scale). and the spectrum of the phase ‘a’ current for, (d) Machine 1, (e) Machine 2, (f) Machine 3.

3.8 Coupled Modeling of the Asymmetrical Triple Star IPM

The triple star machine that was modeled in the section 3.7 is a symmetrical one. Symmetry in the triple star machine means each two adjacent machine have the same phase shift between them. To reduce the harmonic content of the flux linkages the machine can be designed as asymmetrical [152, 153]. In this new configuration the phase shifts between three-phase sets 1 and 2 and also the phase shifts between the three-phase sets 2 and 3 of machine is equal to the half of that in the symmetrical one. Therefore, unlike the symmetrical configuration, the phase shifts between each two adjacent machines are not equal. This machine also is a triple star machine and it also can be considered as three isolated sets of three phase machine. Therefore, this configuration also enjoys the high reliability of the symmetrical one. If any of the machines has a fault, that machine can be removed by putting its stator voltage equal to zero and support the load or turbine using the rest of the machines. The machine is composed of three sets of three phase machines. The machine has 36 slots and each of the machines cover 12 slot of that. The slot angular pitch is:

$$\gamma = \frac{180 \times P}{36} = \frac{180 \times 4}{36} = 20 \ \ (Degree) \tag{3.100}$$

The slots between phases for each set can be calculated as:

$$Slots\ \ Between\ \ Phases = \frac{120}{\gamma} = \frac{120}{20} = 6 \tag{3.101}$$

The full coil pitch is:

$$FCP = \frac{36}{P} = \frac{36}{4} = 9 \tag{3.102}$$

Since the machine has concentrated windings then the belt is equal to 1. And also since the machine is an asymmetrical one the slot between two adjacent machines is [139]:

$$Slot\ \ Betwee\ \ Adjacent\ \ Machines = 0.5 \times \frac{6}{2} = 1 \tag{3.103}$$

For the machine 1 the winding scheme is:

Table 3.5 The winding scheme of machine 1.

$A1^+$	$A1^-$	$B1^+$	$B1^-$	$C1^+$	$C1^-$
1	10	7	16	13	22

$A1_-$	$A1_+$	$B1_-$	$B1_+$	$C1_-$	$C1_+$
10	19	16	25	22	31

$A1^+$	$A1^-$	$B1^+$	$B1^-$	$C1^+$	$C1^-$
19	28	25	34	31	4

$A1_-$	$A1_+$	$B1_-$	$B1_+$	$C1_-$	$C1_+$
28	1	34	7	4	13

The machine 2 has 1 slot shift from the machine 1, therefore the winding scheme for the machine 2 is:

Table 3.6 The winding scheme of machine 2.

$A2^+$	$A2^-$	$B2^+$	$B2^-$	$C2^+$	$C2^-$
2	11	8	17	14	23

$A2_-$	$A2_+$	$B2_-$	$B2_+$	$C2_-$	$C2_+$
11	20	17	26	23	32

$A2^+$	$A2^-$	$B2^+$	$B2^-$	$C2^+$	$C2^-$
20	29	26	35	32	5

$A2_-$	$A2_+$	$B2_-$	$B2_+$	$C2_-$	$C2_+$
29	2	35	8	5	14

The machine 3 has 1 slot shift from the machine 2, therefor the winding scheme for the machine 3 is:

Table 3.7 The winding scheme of machine 3.

$A3^+$	$A3^-$	$B3^+$	$B3^-$	$C3^+$	$C3^-$
3	12	9	18	15	24

$A3_-$	$A3_+$	$B3_-$	$B3_+$	$C3_-$	$C3_+$
12	21	18	27	24	33

$A3^+$	$A3^-$	$B3^+$	$B3^-$	$C3^+$	$C3^-$
21	30	27	36	33	6

$A3_-$	$A3_+$	$B3_-$	$B3_+$	$C3_-$	$C3_+$
30	3	36	9	6	15

Using the above tables, the clock diagram can be drawn as Figure 3.103.

3.8.1 The Model Equations

The machine modelling can proceed from the voltage equations of the stator phases. Phase voltages for phases ‘a’, ‘b’, ‘c’ of each of the three phase IPM machines are given as equation (3.104). In this equation ‘I_{xi}’ is the current of phase ‘x’ and ‘$p\lambda_x$’ is the derivation of the flux linkage seen from the phase ‘x’ of the machine ‘i’ and the term ‘r_s’ also represents the stator resistance for each phase of the stator.

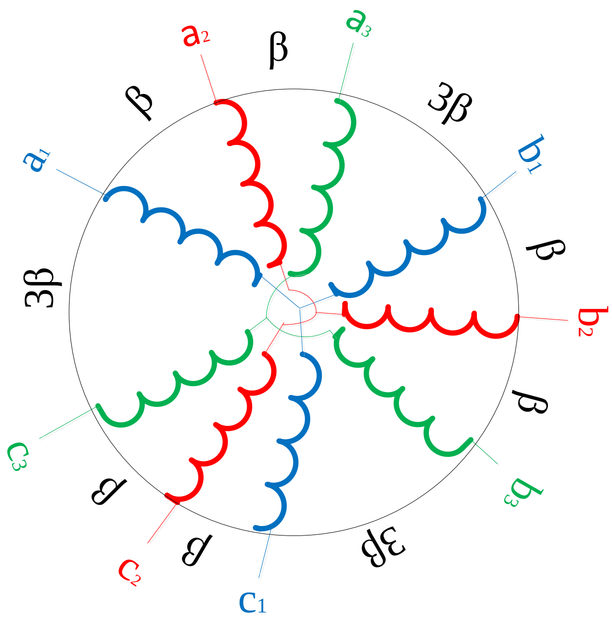


Figure 3.102: The asymmetrical triple star machine connection.

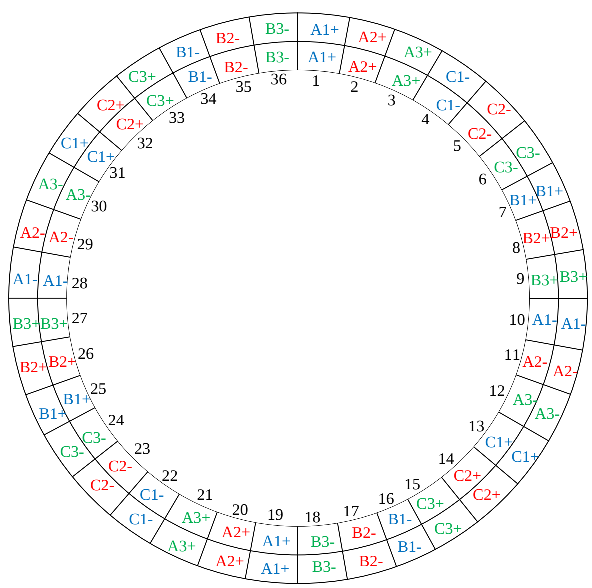


Figure 3.103: The clock diagram of the asymmetrical triple star machine.

$$
\begin{aligned}
V_{ai} &= r_s i_{ai} + p\lambda_{ai} \\
V_{bi} &= r_s i_{bi} + p\lambda_{bi} \quad ,i = 1,2,3 \\
V_{ci} &= r_s i_{ci} + p\lambda_{ci}
\end{aligned}
\tag{3.104}
$$

Therefore, for machine 1 the phase voltage equations are:

$$V_{a1} = r_s i_{a1} + p\lambda_{a1}$$
$$V_{b1} = r_s i_{b1} + p\lambda_{b1} \qquad (3.105)$$
$$V_{c1} = r_s i_{c1} + p\lambda_{c1}$$

And for machine 2 the phase voltage equations are:

$$V_{a2} = r_s i_{a2} + p\lambda_{a2}$$
$$V_{b2} = r_s i_{b2} + p\lambda_{b2} \qquad (3.106)$$
$$V_{c2} = r_s i_{c2} + p\lambda_{c2}$$

And also for machine 3 the phase voltage equations are:

$$V_{a3} = r_s i_{a3} + p\lambda_{a3}$$
$$V_{b3} = r_s i_{b3} + p\lambda_{b3} \qquad (3.107)$$
$$V_{c3} = r_s i_{c3} + p\lambda_{c3}$$

The flux linkage for each phase has four components:

1- The flux linkage due to the current through the phase windings.
2- The flux linkage due to the coupling between the phase and the other phases of the same machine.
3- The flux linkage due to the mutual inductances between the phase and the phases of the other machines.
4- The flux linkage due to the permanent magnets of the rotor.

The flux linkages of the machine phases can be represented as equation (3.108) to (3.110). In this equation the terms 'L_{xixi}' represents the self-inductance of the phase *'x'* of the machine *'i'* and 'L_{xiyj}' represents the mutual inductance between phase 'x' of the machine *'i'* and phase *'y'* of the machine *'j'*. Also the term *'λ_{pmxi}'* represents the flux linkage due to the permanent magnets of the rotor that links the phase *'x'* of the machine *'i'*. Therefore, for the machine 1 the flux linkages are:

$$\begin{aligned}
\lambda_{a1} &= L_{a1a1}i_{a1} + L_{a1b1}i_{b1} + L_{a1c1}i_{c1} + L_{a1a2}i_{a2} + L_{a1b2}i_{b2} + L_{a1c2}i_{c2} + L_{a1a3}i_{a3} + L_{a1b3}i_{b3} + L_{a1c3}i_{c3} + \lambda_{pma1} \\
\lambda_{b1} &= L_{b1a1}i_{a1} + L_{b1b1}i_{b1} + L_{b1c1}i_{c1} + L_{b1a2}i_{a2} + L_{b1b2}i_{b2} + L_{b1c2}i_{c2} + L_{b1a3}i_{a3} + L_{b1b3}i_{b3} + L_{b1c3}i_{c3} + \lambda_{pmb1} \\
\lambda_{c1} &= L_{c1a1}i_{a1} + L_{c1b1}i_{b1} + L_{c1c1}i_{c1} + L_{c1a2}i_{a2} + L_{c1b2}i_{b2} + L_{c1c2}i_{c2} + L_{c1a3}i_{a3} + L_{c1b3}i_{b3} + L_{c1c3}i_{c3} + \lambda_{pmc1}
\end{aligned} \tag{3.108}$$

And for the machine 2 the flux linkages are:

$$\begin{aligned}
\lambda_{a2} &= L_{a2a1}i_{a1} + L_{a2b1}i_{b1} + L_{a2c1}i_{c1} + L_{a2a2}i_{a2} + L_{a2b2}i_{b2} + L_{a2c2}i_{c2} + L_{a1a3}i_{a3} + L_{a2b3}i_{b3} + L_{a2c3}i_{c3} + \lambda_{pma2} \\
\lambda_{b2} &= L_{b2a1}i_{a1} + L_{b2b1}i_{b1} + L_{b2c1}i_{c1} + L_{b2b2}i_{b2} + L_{b2a2}i_{a2} + L_{b2c2}i_{c2} + L_{b2a3}i_{a3} + L_{b2b3}i_{b3} + L_{b2c3}i_{c3} + \lambda_{pmb2} \\
\lambda_{c2} &= L_{c2a1}i_{a1} + L_{c2b1}i_{b1} + L_{c2c1}i_{c1} + L_{c2a2}i_{a2} + L_{c2b2}i_{b2} + L_{c2c2}i_{c2} + L_{c2a3}i_{a3} + L_{c2b3}i_{b3} + L_{c2c3}i_{c3} + \lambda_{pmc2}
\end{aligned} \tag{3.109}$$

And for the machine 3 the flux linkages are:

$$\begin{aligned}
\lambda_{a3} &= L_{a3a1}i_{a1} + L_{a3b1}i_{b1} + L_{a3c1}i_{c1} + L_{a3a2}i_{a2} + L_{a3b2}i_{b2} + L_{a3c2}i_{c2} + L_{a3a3}i_{a3} + L_{a3b3}i_{b3} + L_{a3c3}i_{c3} + \lambda_{pma3} \\
\lambda_{b3} &= L_{b3a1}i_{a1} + L_{b3b1}i_{b1} + L_{b3c1}i_{c1} + L_{b3b2}i_{b2} + L_{b3a2}i_{a2} + L_{b3c2}i_{c2} + L_{b3a3}i_{a3} + L_{b3b3}i_{b3} + L_{b3c3}i_{c3} + \lambda_{pmb3} \\
\lambda_{c3} &= L_{c3a1}i_{a1} + L_{c3b1}i_{b1} + L_{c3c1}i_{c1} + L_{c3a2}i_{a2} + L_{c3b2}i_{b2} + L_{c3c2}i_{c2} + L_{c3a3}i_{a3} + L_{c3b3}i_{b3} + L_{c3c3}i_{c3} + \lambda_{pmc3}
\end{aligned} \tag{3.110}$$

The flux linkages can be represented in the matrix form as:

$$\lambda_{abci} = L_s i_{sabci} + \lambda_{Pmabci} =$$

$$\begin{pmatrix} L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} & L_{a1a3} & L_{a1b3} & L_{a1c3} \\ L_{b1a1} & L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} & L_{b1a3} & L_{b1b3} & L_{b1c3} \\ L_{c1a1} & L_{c1b1} & L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} & L_{c1a3} & L_{c1b3} & L_{c1c3} \\ L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{a2a2} & L_{a2b2} & L_{a2c2} & L_{a2a3} & L_{a2b3} & L_{a2c3} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{b2b2} & L_{b2c2} & L_{b2a3} & L_{b2b3} & L_{b2c3} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{c2c2} & L_{c2a3} & L_{c2b3} & L_{c2c3} \\ L_{a3a1} & L_{a3b1} & L_{a3c1} & L_{a3a2} & L_{a3b2} & L_{a3c2} & L_{a3a3} & L_{a3b3} & L_{a3c3} \\ L_{b3a1} & L_{b3b1} & L_{b3c1} & L_{b3a2} & L_{b3b2} & L_{b3c2} & L_{b3a3} & L_{b3b3} & L_{b3c3} \\ L_{c3a1} & L_{c3b1} & L_{c3c1} & L_{c3a2} & L_{c3b2} & L_{c3c2} & L_{c3a3} & L_{c3b3} & L_{c3c3} \end{pmatrix} \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \\ i_{a3} \\ i_{b3} \\ i_{c3} \end{bmatrix} + \begin{bmatrix} \lambda_{pma1} \\ \lambda_{pmb1} \\ \lambda_{pmc1} \\ \lambda_{pma2} \\ \lambda_{pmb2} \\ \lambda_{pmc2} \\ \lambda_{pma3} \\ \lambda_{pmb3} \\ \lambda_{pmc3} \end{bmatrix} \quad (3.111)$$

The transformation matrix of equation (3.112) can be used to transform the voltage equations to the rotor reference frame. This transformation has three parts. Each part is aligned to the phase ‘a’ of each of the machines. Therefore, the top part is aligned to the machine ‘1” the middle one is aligned with the machine ‘2’ and the bottom one is for machine ‘3’[83].

$T(\theta_r) =$

$$
\begin{pmatrix}
C(\theta_r) & C(\theta_r-\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\
S(\theta_r) & S(\theta_r-\gamma) & S(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & C(\theta_r-\beta) & C(\theta_r-\gamma-\beta) & C(\theta_r+\gamma-\beta) & 0 & 0 & 0 \\
0 & 0 & 0 & S(\theta_r-\beta) & S(\theta_r-\gamma-\beta) & S(\theta_r+\gamma-\beta) & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta) & C(\theta_r-2\beta-\gamma) & C(\theta_r-2\beta+\gamma) \\
0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r-2\beta) & S(\theta_r-2\beta-\gamma) & S(\theta_r-2\beta+\gamma) \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix} \qquad (3.112)
$$

$\beta = \frac{\pi}{9},\ \gamma = \frac{2\pi}{3},\ \theta_r = Rotor\ \ Angle$

Where: $\gamma = 2\pi/3$, which is the phase shift between two adjacent phases of each machine set, $\beta = \pi/9$ which is the phase shift between two adjacent machines and θ_r is the rotor angle. Also in this matrix 'C' represents 'cos' and 'S' represents 'sin'.

The currents and flux linkages of equation (3.104) can be replaced by their corresponding currents in rotor reference frame according to equation (3.113).

$$V_{xis} = r_{xis} T(\theta_r)^{-1} i_{qdr} + p\left[T(\theta_r)^{-1} \lambda_{qdr}\right] \tag{3.113}$$

Multiplying the $T(\theta_r)$ from the left side of the equation, results in:

$$T(\theta_r) V_{xis} = T(\theta_r) r_{xis} T(\theta_r)^{-1} i_{qdor} + T(\theta_r) p\left[T(\theta_r)^{-1} \lambda_{qdor}\right] \tag{3.114}$$

This can be rewritten as:

$$V_{qdor} = T(\theta_r) r_{xis} T(\theta_r)^{-1} i_{qdor} + T(\theta_r) p\left[T(\theta_r)^{-1} \lambda_{qdor}\right] \tag{3.115}$$

The different terms of the equation (3.115) can be represented as:

$$T(\theta_r) r_{xs} T(\theta_r)^{-1} = \begin{pmatrix} r_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_s \end{pmatrix} \tag{3.116}$$

$$T(\theta_r) p\left[T(\theta_r)^{-1} \lambda_{qdor}\right] = T(\theta_r) p T(\theta_r)^{-1} \lambda_{qdor} + T(\theta_r) T(\theta_r)^{-1} p \lambda_{qdor} \tag{3.117}$$

In equation (3.117) the term with derivation of the transformation matrix can be written as equation (3.118).

$$T(\theta_r)pT(\theta_r)^{-1} =$$

$$\frac{2}{3}\begin{pmatrix} C(\theta_r) & C(\theta_r-\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ S(\theta_r) & S(\theta_r-\gamma) & S(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r-\gamma) & C(\theta_r-\gamma-\beta) & C(\theta_r+\gamma-\beta) & 0 & 0 & 0 \\ 0 & 0 & 0 & S(\theta_r-\gamma) & S(\theta_r-\gamma-\beta) & S(\theta_r+\gamma-\beta) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta) & C(\theta_r-2\beta-\gamma) & C(\theta_r-2\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r-2\beta) & S(\theta_r-2\beta-\gamma) & S(\theta_r-2\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \times \tag{3.118}$$

$$p\begin{pmatrix} C(\theta_r) & S(\theta_r) & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ C(\theta_r-\gamma) & S(\theta_r-\gamma) & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ C(\theta_r+\gamma) & S(\theta_r+\gamma) & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r-\beta) & S(\theta_r-\beta) & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r-\gamma-\beta) & S(\theta_r-\gamma-\beta) & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r+\gamma-\beta) & S(\theta_r+\gamma-\beta) & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta) & S(\theta_r-2\beta) & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta-\gamma) & S(\theta_r-2\beta-\gamma) & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta+\gamma) & S(\theta_r-2\beta+\gamma) & \frac{1}{2} \end{pmatrix}$$

Substituting the derivation of the second matrix in to the above equation results in:

$$T(\theta_r)pT(\theta_r)^{-1} =$$

$$\begin{pmatrix}
C(\theta_r) & C(\theta_r-\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\
S(\theta_r) & S(\theta_r-\gamma) & S(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & C(\theta_r-\gamma) & C(\theta_r-\gamma-\beta) & C(\theta_r+\gamma-\beta) & 0 & 0 & 0 \\
0 & 0 & 0 & S(\theta_r-\gamma) & S(\theta_r-\gamma-\beta) & S(\theta_r+\gamma-\beta) & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta) & C(\theta_r-2\beta-\gamma) & C(\theta_r-2\beta+\gamma) \\
0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r-2\beta) & S(\theta_r-2\beta-\gamma) & S(\theta_r-2\beta+\gamma) \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix} \times \tag{3.119}$$

$$\frac{2\omega_r}{3}\begin{pmatrix}
-S(\theta_r) & C(\theta_r) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-S(\theta_r-\gamma) & C(\theta_r-\gamma) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-S(\theta_r+\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -S(\theta_r-\beta) & C(\theta_r-\beta) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -S(\theta_r-\gamma-\beta) & C(\theta_r-\gamma-\beta) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -S(\theta_r+\gamma-\beta) & C(\theta_r+\gamma-\beta) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -S(\theta_r-2\beta) & C(\theta_r-2\beta) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -S(\theta_r-2\beta-\gamma) & C(\theta_r-2\beta-\gamma) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -S(\theta_r-2\beta+\gamma) & C(\theta_r-2\beta+\gamma) & 0
\end{pmatrix} = \omega_r\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

The flux linkage of the machine can also be presented as:

$$\lambda_{qdor} = T(\theta_r) L_{ss} \boldsymbol{i}_{abci} + T(\theta_r) \lambda_{pm\_abci} \tag{3.120}$$

Substituting the currents by their corresponding currents in rotor reference frame results in:

$$\lambda_{qdor} = T(\theta_r) L_{ss} T^{-1}(\theta_r) \boldsymbol{i}_{qdor} + T(\theta_r) \lambda_{pm\_abci} \tag{3.121}$$

The flux linkage due to the permanent magnet of the machine in the stator phases can be transformed to the rotor reference frame according to equation (3.122).

$$T(\theta_r)\lambda_{pm\_abci} =$$

$$\frac{2}{3}\begin{pmatrix} C(\theta_r) & C(\theta_r-\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ S(\theta_r) & S(\theta_r-\gamma) & S(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r-\gamma) & C(\theta_r-\gamma-\beta) & C(\theta_r+\gamma-\beta) & 0 & 0 & 0 \\ 0 & 0 & 0 & S(\theta_r-\gamma) & S(\theta_r-\gamma-\beta) & S(\theta_r+\gamma-\beta) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta) & C(\theta_r-2\beta-\gamma) & C(\theta_r-2\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r-2\beta) & S(\theta_r-2\beta-\gamma) & S(\theta_r-2\beta+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \times \tag{3.122}$$

$$\begin{bmatrix} \lambda_{pm\_a1} \\ \lambda_{pm\_b1} \\ \lambda_{pm\_c1} \\ \lambda_{pm\_a2} \\ \lambda_{pm\_b2} \\ \lambda_{pm\_c2} \\ \lambda_{pm\_a3} \\ \lambda_{pm\_b3} \\ \lambda_{pm\_c3} \end{bmatrix} = \begin{bmatrix} \lambda_{pmq1r} \\ \lambda_{pmd1r} \\ \lambda_{pmo1r} \\ \lambda_{pmd2r} \\ \lambda_{pmq2r} \\ \lambda_{pmo2r} \\ \lambda_{pmq3r} \\ \lambda_{pmd3r} \\ \lambda_{pmo3r} \end{bmatrix}$$

The equation (3.121) also has a term which includes 'L_{ss}' representing the inductances of the stator of the machine. The inductances can be transformed to the rotor reference frame as equation (3.123).

$$
\begin{aligned}
&T(\theta_r)L_{ss}T(\theta_r)^{-1} = \\
&\frac{2}{3}\begin{pmatrix}
C(\theta_r) & C(\theta_r-\gamma) & C(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\
S(\theta_r) & S(\theta_r-\gamma) & S(\theta_r+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & C(\theta_r-\gamma) & C(\theta_r-\gamma-\beta) & C(\theta_r+\gamma-\beta) & 0 & 0 & 0 \\
0 & 0 & 0 & S(\theta_r-\gamma) & S(\theta_r-\gamma-\beta) & S(\theta_r+\gamma-\beta) & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta) & C(\theta_r-2\beta-\gamma) & C(\theta_r-2\beta+\gamma) \\
0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r-2\beta) & S(\theta_r-2\beta-\gamma) & S(\theta_r-2\beta+\gamma) \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}\times \\
&\begin{pmatrix}
L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} & L_{a1a3} & L_{a1b3} & L_{a1c3} \\
L_{b1a1} & L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} & L_{b1a3} & L_{b1b3} & L_{b1c3} \\
L_{c1a1} & L_{c1b1} & L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} & L_{c1a3} & L_{c1b3} & L_{c1c3} \\
L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{a2a2} & L_{a2b2} & L_{a2c2} & L_{a2a3} & L_{a2b3} & L_{a2c3} \\
L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{b2b2} & L_{b2c2} & L_{b2a3} & L_{b2b3} & L_{b2c3} \\
L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{c2c2} & L_{c2a3} & L_{c2b3} & L_{c2c3} \\
L_{a3a1} & L_{a3b1} & L_{a3c1} & L_{a3a2} & L_{a3b2} & L_{a3c2} & L_{a3a3} & L_{a3b3} & L_{a3c3} \\
L_{b3a1} & L_{b3b1} & L_{b3c1} & L_{b3a2} & L_{b3b2} & L_{b3c2} & L_{b3a3} & L_{b3b3} & L_{b3c3} \\
L_{c3a1} & L_{c3b1} & L_{c3c1} & L_{c3a2} & L_{c3b2} & L_{c3c2} & L_{c3a3} & L_{c3b3} & L_{c3c3}
\end{pmatrix}\times \\
&\begin{pmatrix}
C(\theta_r) & S(\theta_r) & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
C(\theta_r-\gamma) & S(\theta_r-\gamma) & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
C(\theta_r+\gamma) & S(\theta_r+\gamma) & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & C(\theta_r-\beta) & S(\theta_r-\beta) & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & C(\theta_r-\gamma-\beta) & S(\theta_r-\gamma-\beta) & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & C(\theta_r+\gamma-\beta) & S(\theta_r+\gamma-\beta) & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta) & S(\theta_r-2\beta) & \frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta-\gamma) & S(\theta_r-2\beta-\gamma) & \frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r-2\beta+\gamma) & S(\theta_r-2\beta+\gamma) & \frac{1}{2}
\end{pmatrix} = \\
&\begin{pmatrix}
L_{q11} & L_{q1d1} & L_{q101} & L_{q1q2} & L_{q1d2} & L_{q102} & L_{q1q3} & L_{q1d3} & L_{q103} \\
L_{d1q1} & L_{d11} & L_{d101} & L_{d1q2} & L_{d1d2} & L_{d102} & L_{d1q3} & L_{d1d3} & L_{d103} \\
L_{01q1} & L_{01d1} & L_{0101} & L_{01q2} & L_{01d2} & L_{0102} & L_{01q3} & L_{01d3} & L_{0103} \\
L_{q1q2} & L_{q1d2} & L_{q102} & L_{q22} & L_{q2d2} & L_{q202} & L_{q2q3} & L_{q2d3} & L_{q203} \\
L_{d1q2} & L_{d1d2} & L_{d102} & L_{d2q2} & L_{d22} & L_{d202} & L_{d2q3} & L_{d2d3} & L_{d203} \\
L_{01q2} & L_{01d2} & L_{0102} & L_{02q2} & L_{02d2} & L_{0202} & L_{02q3} & L_{02d3} & L_{0203} \\
L_{q1q3} & L_{q1d3} & L_{q103} & L_{q2q3} & L_{q2d3} & L_{q203} & L_{q33} & L_{q3d3} & L_{q303} \\
L_{d1q3} & L_{d1d3} & L_{d103} & L_{d2q3} & L_{d2d3} & L_{d203} & L_{d3q3} & L_{d33} & L_{d303} \\
L_{01q3} & L_{01d3} & L_{0103} & L_{02q3} & L_{02d3} & L_{0203} & L_{03q3} & L_{03d3} & L_{0303}
\end{pmatrix}
\end{aligned}
\tag{3.123}
$$

Then using the permanent magnet flux linkages and the inductances of the machine in the rotor reference frame, the flux linkages of the machines in equation (3.123), can be represented in rotor reference frame as equation (3.124).

$$\lambda_{q1} = L_{q1q1} i_{q1} + L_{q1d1} i_{d1} + L_{q1o1} i_{o1} + L_{q1q2} i_{q2} + L_{q1d2} i_{d2} + L_{q1o2} i_{o2} + L_{q1q3} i_{q3} + L_{q1d3} i_{d3} + L_{q1o3} i_{o3} + \lambda_{pmq1}$$

$$\lambda_{d1} = L_{d1d1} i_{d1} + L_{d1q1} i_{q1} + L_{d1o1} i_{o1} + L_{d1d2} i_{d2} + L_{d1q2} i_{q2} + L_{d1o2} i_{o2} + L_{d1d3} i_{d3} + L_{d1q3} i_{q3} + L_{d1o3} i_{o3} + \lambda_{pmd1}$$

$$\lambda_{o1} = L_{o1d1} i_{d1} + L_{o1q1} i_{q1} + L_{o1o1} i_{o1} + L_{o1d2} i_{d2} + L_{o1q2} i_{q2} + L_{o1o2} i_{o2} + L_{o1d3} i_{d3} + L_{o1q3} i_{q3} + L_{o1o3} i_{o3} + \lambda_{pmo1}$$

$$\lambda_{q2} = L_{q2q2} i_{q2} + L_{q2d2} i_{d2} + L_{q2o2} i_{o2} + L_{q2q1} i_{q1} + L_{q2d1} i_{d1} + L_{q2o1} i_{o1} + L_{q2q3} i_{q3} + L_{q2d3} i_{d3} + L_{q2o3} i_{o3} + \lambda_{pmq2}$$

$$\lambda_{d2} = L_{d2d2} i_{d2} + L_{d2q2} i_{q2} + L_{d2o2} i_{o2} + L_{d2d1} i_{d1} + L_{d2q1} i_{q1} + L_{d2o1} i_{o1} + L_{d2d3} i_{d3} + L_{d2q3} i_{q3} + L_{d2o3} i_{o3} + \lambda_{pmd2} \quad (3.124)$$

$$\lambda_{o2} = L_{o2d2} i_{d2} + L_{o2q2} i_{q2} + L_{o2o2} i_{o2} + L_{o2d1} i_{d1} + L_{o2q1} i_{q1} + L_{o2o1} i_{o1} + L_{o2d3} i_{d3} + L_{o2q3} i_{q3} + L_{o2o3} i_{o3} + \lambda_{pmo2}$$

$$\lambda_{q3} = L_{q3q3} i_{q3} + L_{q3d3} i_{d3} + L_{q3o3} i_{o3} + L_{q3q1} i_{q1} + L_{q3d1} i_{d1} + L_{q3o1} i_{o1} + L_{q3q2} i_{q2} + L_{q3d2} i_{d2} + L_{q3o2} i_{o2} + \lambda_{pmq3}$$

$$\lambda_{d3} = L_{d3d3} i_{d3} + L_{d3q3} i_{q3} + L_{d3o3} i_{o3} + L_{d3d1} i_{d1} + L_{d3q1} i_{q1} + L_{d3o1} i_{o1} + L_{d3d2} i_{d2} + L_{d3q2} i_{q2} + L_{d3o2} i_{o2} + \lambda_{pmd3}$$

$$\lambda_{o3} = L_{o3d3} i_{d3} + L_{o3q3} i_{q3} + L_{o3o3} i_{o3} + L_{o3d1} i_{d1} + L_{o3q1} i_{q1} + L_{o3o1} i_{o1} + L_{o3d2} i_{d2} + L_{o3q2} i_{q2} + L_{o3o2} i_{o2} + \lambda_{pmo3}$$

Also the voltages and the currents of the machines can be transformed to the rotor reference frame as equation (3.125) and (3.126) respectively.

$$\begin{bmatrix} V_{q1} \\ V_{d1} \\ V_{01} \\ V_{q2} \\ V_{d2} \\ V_{02} \\ V_{q3} \\ V_{d3} \\ V_{03} \end{bmatrix} = T_1(\theta_r)V_{s123} = T_1(\theta_r) \times \begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \\ V_{a2} \\ V_{b2} \\ V_{c2} \\ V_{a3} \\ V_{b3} \\ V_{c3} \end{bmatrix} \tag{3.125}$$

$$\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} = T_1(\theta_r)i_{s123} = T_1(\theta_r) \times \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \\ i_{a3} \\ i_{b3} \\ i_{c3} \end{bmatrix} \tag{3.126}$$

By separating the voltages of q and d axis of different machines, the voltage equations of the machines could be represented as:

$$\begin{aligned} V_{q1} &= r_s i_{q1} + \omega_r \lambda_{d1} + p\lambda_{q1} \\ V_{d1} &= r_s i_{d1} - \omega_r \lambda_{q1} + p\lambda_{d1} \\ V_{01} &= r_s i_{01} + p\lambda_{01} \end{aligned} \tag{3.127}$$

$$V_{q2} = r_s i_{q2} + \omega_r \lambda_{d2} + p\lambda_{q2}$$
$$V_{d2} = r_s i_{d2} - \omega_r \lambda_{q2} + p\lambda_{d2} \qquad (3.128)$$
$$V_{02} = r_s i_{02} + p\lambda_{02}$$

$$V_{q3} = r_s i_{q3} + \omega_r \lambda_{d3} + p\lambda_{q3}$$
$$V_{d3} = r_s i_{d3} - \omega_r \lambda_{q3} + p\lambda_{d3} \qquad (3.129)$$
$$V_{03} = r_s i_{03} + p\lambda_{03}$$

The generated torque of each machine can also be calculated using the co-energy equation. The co-energy of the machine 1 can be presented as function of the stator currents and the flux linkages as:

$$W_{co1} = \frac{1}{2} i_{abc1}{}^{t} L_{ss} i_{abc1,2,3} + i_{s1}{}^{t} \lambda_{pm\_1} \qquad (3.130)$$

From the co-energy equation, the electromagnetic torque can be derived as:

$$T_{e1} = \frac{\partial W_{co1}}{\partial \theta_{rm}} \qquad (3.131)$$

The equation (3.130) is equal to:

$$T_{e1} = \frac{1}{2} i_{abc1}{}^{t} \frac{\partial L_{ss}}{\partial \theta_{rm}} i_{abc1,2,3} + i_{abc1}{}^{t} \frac{\partial \lambda_{pm\_1}}{\partial \theta_{rm}} \qquad (3.132)$$

Since the previous equations are in term of the electrical angle, the mechanical angle of the equation (3.132) needs to be converted to the electrical equation as equation (3.133).

$$\theta_r = \frac{P}{2} \theta_{rm} \qquad (3.133)$$

Therefore, the torque equation changes to equation (3.134).

$$T_{e1} = \frac{P}{2}\frac{1}{2} i_{abc1}{}^{t} \frac{\partial L_{ss}}{\partial \theta_r} i_{abc1,2,3} + \frac{P}{2} i_{s1}{}^{t} \frac{\partial \lambda_{pm\_1}}{\partial \theta_r} \tag{3.134}$$

Substituting the stator currents with their corresponding values in rotor reference frame results in:

$$T_{e1} = \frac{3}{2}\frac{P}{2} (i_{qdo1})^{t} \, T(\theta_r) \frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} i_{qdo1,2,3} + \frac{3}{2}\frac{P}{2} (i_{qd1})^{t} \, T(\theta_r) \frac{\partial \lambda_{pm\_1}}{\partial \theta_r} \tag{3.135}$$

The equation can be rewritten as:

$$T_{e1} = \frac{3}{2}\frac{P}{2} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} T(\theta_r) \frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} T(\theta_r) \frac{\partial \lambda_{pm}}{\partial \theta_r} \tag{3.136}$$

This equation is equal to:

$$T_{e1} = \frac{3}{2}\frac{P}{2} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t} L_{qdo} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{t^{t}} \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix} \tag{3.137}$$

Replacing the inductance matrix from the equation (3.123) results in:

$$T_{e1} =$$

$$\frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{pmatrix} L_{q11} & L_{q1d1} & L_{q101} & L_{q1q2} & L_{q1d2} & L_{q102} & L_{q1q3} & L_{q1d3} & L_{q103} \\ L_{d1q1} & L_{d11} & L_{d101} & L_{d1q2} & L_{d1d2} & L_{d102} & L_{d1q3} & L_{d1d3} & L_{d103} \\ L_{01q1} & L_{01d1} & L_{0101} & L_{01q2} & L_{01d2} & L_{0102} & L_{01q3} & L_{01d3} & L_{0103} \\ L_{q1q2} & L_{q1d2} & L_{q102} & L_{q22} & L_{q2d2} & L_{q202} & L_{q2q3} & L_{q2d3} & L_{q203} \\ L_{d1q2} & L_{d1d2} & L_{d102} & L_{d2q2} & L_{d22} & L_{d202} & L_{d2q3} & L_{d2d3} & L_{d203} \\ L_{01q2} & L_{01d2} & L_{0102} & L_{02q2} & L_{02d2} & L_{0202} & L_{02q3} & L_{02d3} & L_{0203} \\ L_{q1q3} & L_{q1d3} & L_{q103} & L_{q2q3} & L_{q2d3} & L_{q203} & L_{q33} & L_{q3d3} & L_{q303} \\ L_{d1q3} & L_{d1d3} & L_{d103} & L_{d2q3} & L_{d2d3} & L_{d203} & L_{d3q3} & L_{d33} & L_{d303} \\ L_{01q3} & L_{01d3} & L_{0103} & L_{02q3} & L_{02d3} & L_{0203} & L_{03q3} & L_{03d3} & L_{0303} \end{pmatrix} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} +$$

$$\frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix} \tag{3.138}$$

The equation (3.138) is equal to:

$$T_{e1} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{pmatrix} L_{q11}i_{q1} + L_{q1d1}i_{d1} + L_{q101}i_{01} + L_{q1q2}i_{q2} + L_{q1d2}i_{d2} + L_{q102}i_{02} + L_{q1q3}i_{q3} + L_{q1d3}i_{d3} + L_{q103}i_{03} \\ L_{d1q1}i_{q1} + L_{d11}i_{d1} + L_{d101}i_{01} + L_{d1q2}i_{q2} + L_{d1d2}i_{d2} + L_{d102}i_{02} + L_{d1q3}i_{q3} + L_{d1d3}i_{d3} + L_{d103}i_{03} \\ L_{01q1}i_{q1} + L_{01d1}i_{d1} + L_{0101}i_{01} + L_{01q2}i_{q2} + L_{01d2}i_{d2} + L_{0102}i_{02} + L_{01q3}i_{q3} + L_{01d3}i_{d3} + L_{0103}i_{03} \\ L_{q1q2}i_{q1} + L_{q1d2}i_{d1} + L_{q102}i_{01} + L_{q22}i_{q2} + L_{q2d2}i_{d2} + L_{q202}i_{02} + L_{q2q3}i_{q3} + L_{q2d3}i_{d3} + L_{q203}i_{03} \\ L_{d1q2}i_{q1} + L_{d1d2}i_{d1} + L_{d102}i_{01} + L_{d2q2}i_{q2} + L_{d22}i_{d2} + L_{d202}i_{02} + L_{d2q3}i_{q3} + L_{d2d3}i_{d3} + L_{d203}i_{03} \\ L_{01q2}i_{q1} + L_{01d2}i_{d1} + L_{0102}i_{01} + L_{02q2}i_{q2} + L_{02d2}i_{d2} + L_{0202}i_{02} + L_{02q3}i_{q3} + L_{02d3}i_{d3} + L_{0203}i_{03} \\ L_{q1q3}i_{q1} + L_{q1d3}i_{d1} + L_{q103}i_{01} + L_{q2q3}i_{q2} + L_{q2d3}i_{d2} + L_{q203}i_{02} + L_{q3q3}i_{q3} + L_{q3d3}i_{d3} + L_{q303}i_{03} \\ L_{d1q3}i_{q1} + L_{d1d3}i_{d1} + L_{d103}i_{01} + L_{d2q3}i_{q2} + L_{d2d3}i_{d2} + L_{d203}i_{02} + L_{d3q3}i_{q3} + L_{d3d3}i_{d3} + L_{d303}i_{03} \\ L_{01q3}i_{q1} + L_{01d3}i_{d1} + L_{0103}i_{01} + L_{02q3}i_{q2} + L_{02d2}i_{d2} + L_{0203}i_{02} + L_{03q3}i_{q3} + L_{03d3}i_{d3} + L_{0303}i_{03} \end{pmatrix} \quad (3.139)$$

$$+\frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix}$$

The non-zero terms of the equation (3.139) are:

$$T_{e1} = \frac{3}{2}\frac{P}{2}\left[\begin{array}{l} \left(\begin{array}{l} L_{q11}i_{q1} + L_{q1d1}i_{d1} + L_{q101}i_{01} + L_{q1q2}i_{q2} + L_{q1d2}i_{d2} + \\ L_{q102}i_{02} + L_{q1q3}i_{q3} + L_{q1d3}i_{d3} + L_{q103}i_{03} \end{array}\right) i_{d1} - \\ \left(\begin{array}{l} L_{d1q1}i_{q1} + L_{d11}i_{d1} + L_{d101}i_{01} + L_{d1q2}i_{q2} + L_{d1d2}i_{d2} + \\ L_{d102}i_{02} + L_{d1q3}i_{q3} + L_{d1d3}i_{d3} + L_{d103}i_{03} \end{array}\right) i_{q1} + \\ \left(\begin{array}{l} L_{01q1}i_{q1} + L_{01d1}i_{d1} + L_{0101}i_{01} + L_{01q2}i_{q2} + L_{01d2}i_{d2} + \\ L_{0102}i_{02} + L_{01q3}i_{q3} + L_{01d3}i_{d3} + L_{0103}i_{03} \end{array}\right) i_{01} \end{array}\right] \tag{3.140}$$

$$+ \frac{3}{2}\frac{P}{2}\left(\lambda_{pmq1}i_{d1} + \lambda_{pmd1}i_{q1} + \lambda_{pm01}i_{01}\right)$$

The co-energy of the machine 2 also can be presented as function of the stator currents and the flux linkages as:

$$W_{co2} = \frac{1}{2} i_{abc2}{}^{t} L_{ss} i_{abc1,2,3} + i_{s1}{}^{t} \lambda_{pm\_2} \tag{3.141}$$

From the torque co-energy equation, the electromagnetic torque can be derived as:

$$T_{e2} = \frac{\partial W_{co2}}{\partial \theta_{rm}} \tag{3.142}$$

The equation (3.142) is equal to:

$$T_{e2} = \frac{1}{2} i_{abc2}{}^{t} \frac{\partial L_{ss}}{\partial \theta_{rm}} i_{abc1,2,3} + i_{abc2}{}^{t} \frac{\partial \lambda_{pm\_2}}{\partial \theta_{rm}} \tag{3.143}$$

Again the mechanical angle of the equation (3.143) needs to be converted to the electrical angle as equation (3.144).

$$\theta_r = \frac{P}{2}\theta_{rm} \tag{3.144}$$

Therefore, the torque equation changes to:

$$T_{e2} = \frac{P}{2}\frac{1}{2} i_{abc2}{}^{t} \frac{\partial L_{ss}}{\partial \theta_r} i_{abc1,2,3} + \frac{P}{2} i_{s2}{}^{t} \frac{\partial \lambda_{pm\_2}}{\partial \theta_r} \tag{3.145}$$

Substituting the stator currents with their corresponding values in rotor reference frame results in:

$$T_{e2} = \frac{3}{2}\frac{P}{2}(i_{qd2})^t\, T(\theta_r)\frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} i_{qdo1,2,3} + \frac{3}{2}\frac{P}{2}(i_{qd2})^t\, T(\theta_r)\frac{\partial \lambda_{pm\_2}}{\partial \theta_r} \tag{3.146}$$

The equation can be rewritten as:

$$T_{e2} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^t L_{qdo}\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix} \tag{3.147}$$

Replacing the qdo inductance matrix in to the equation (3.147) results in:

$$T_{e2} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{pmatrix} L_{q11} & L_{q1d1} & L_{q101} & L_{q1q2} & L_{q1d2} & L_{q102} & L_{q1q3} & L_{q1d3} & L_{q103} \\ L_{d1q1} & L_{d11} & L_{d101} & L_{d1q2} & L_{d1d2} & L_{d102} & L_{d1q3} & L_{d1d3} & L_{d103} \\ L_{01q1} & L_{01d1} & L_{0101} & L_{01q2} & L_{01d2} & L_{0102} & L_{01q3} & L_{01d3} & L_{0103} \\ L_{q1q2} & L_{q1d2} & L_{q102} & L_{q22} & L_{q2d2} & L_{q202} & L_{q2q3} & L_{q2d3} & L_{q203} \\ L_{d1q2} & L_{d1d2} & L_{d102} & L_{d2q2} & L_{d22} & L_{d202} & L_{d2q3} & L_{d2d3} & L_{d203} \\ L_{01q2} & L_{01d2} & L_{0102} & L_{02q2} & L_{02d2} & L_{0202} & L_{02q3} & L_{02d3} & L_{0203} \\ L_{q1q3} & L_{q1d3} & L_{q103} & L_{q2q3} & L_{q2d3} & L_{q203} & L_{q33} & L_{q3d3} & L_{q303} \\ L_{d1q3} & L_{d1d3} & L_{d103} & L_{d2q3} & L_{d2d3} & L_{d203} & L_{d3q3} & L_{d33} & L_{d303} \\ L_{01q3} & L_{01d3} & L_{0103} & L_{02q3} & L_{02d3} & L_{0203} & L_{03q3} & L_{03d3} & L_{0303} \end{pmatrix} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} +$$

$$\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix} \tag{3.148}$$

This equation is equal to:

$$T_{e2} =$$

$$\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{pmatrix} L_{q11}i_{q1} + L_{q1d1}i_{d1} + L_{q101}i_{01} + L_{q1q2}i_{q2} + L_{q1d2}i_{d2} + L_{q102}i_{02} + L_{q1q3}i_{q3} + L_{q1d3}i_{d3} + L_{q103}i_{03} \\ L_{d1q1}i_{q1} + L_{d11}i_{d1} + L_{d101}i_{01} + L_{d1q2}i_{q2} + L_{d1d2}i_{d2} + L_{d102}i_{02} + L_{d1q3}i_{q3} + L_{d1d3}i_{d3} + L_{d103}i_{03} \\ L_{01q1}i_{q1} + L_{01d1}i_{d1} + L_{0101}i_{01} + L_{01q2}i_{q2} + L_{01d2}i_{d2} + L_{0102}i_{02} + L_{01q3}i_{q3} + L_{01d3}i_{d3} + L_{0103}i_{03} \\ L_{q1q2}i_{q1} + L_{q1d2}i_{d1} + L_{q102}i_{01} + L_{q22}i_{q2} + L_{q2d2}i_{d2} + L_{q202}i_{02} + L_{q2q3}i_{q3} + L_{q2d3}i_{d3} + L_{q203}i_{03} \\ L_{d1q2}i_{q1} + L_{d1d2}i_{d1} + L_{d102}i_{01} + L_{d2q2}i_{q2} + L_{d22}i_{d2} + L_{d202}i_{02} + L_{d2q3}i_{q3} + L_{d2d3}i_{d3} + L_{d203}i_{03} \\ L_{01q2}i_{q1} + L_{01d2}i_{d1} + L_{0102}i_{01} + L_{02q2}i_{q2} + L_{02d2}i_{d2} + L_{0202}i_{02} + L_{02q3}i_{q3} + L_{02d3}i_{d3} + L_{0203}i_{03} \\ L_{q1q3}i_{q1} + L_{q1d3}i_{d1} + L_{q103}i_{01} + L_{q2q3}i_{q2} + L_{q2d3}i_{d2} + L_{q203}i_{02} + L_{q3q3}i_{q3} + L_{q3d3}i_{d3} + L_{q303}i_{03} \\ L_{d1q3}i_{q1} + L_{d1d3}i_{d1} + L_{d103}i_{01} + L_{d2q3}i_{q2} + L_{d2d3}i_{d2} + L_{d203}i_{02} + L_{d3q3}i_{q3} + L_{d3d3}i_{d3} + L_{d303}i_{03} \\ L_{01q3}i_{q1} + L_{01d3}i_{d1} + L_{0103}i_{01} + L_{02q3}i_{q2} + L_{02d2}i_{d2} + L_{0203}i_{02} + L_{03q3}i_{q3} + L_{03d3}i_{d3} + L_{0303}i_{03} \end{pmatrix} \quad (3.149)$$

$$+\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ i_{q2} \\ i_{d2} \\ i_{02} \\ 0 \\ 0 \\ 0 \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix}$$

The non-zero terms of the above equation are:

$T_{e2} =$

$$\frac{3}{2}\frac{P}{2}\left[\begin{aligned}&\left(\begin{aligned}&L_{q1q2}i_{q1}+L_{q1d2}i_{d1}+L_{q102}i_{01}+L_{q22}i_{q2}+L_{q2d2}i_{d2}+\\&L_{q202}i_{02}+L_{q2q3}i_{q3}+L_{q2d3}i_{d3}+L_{q203}i_{03}\end{aligned}\right)i_{d2}-\\&\left(\begin{aligned}&L_{d1q2}i_{q1}+L_{d1d2}i_{d1}+L_{d102}i_{01}+L_{d2q2}i_{q2}+L_{d22}i_{d2}+\\&L_{d202}i_{02}+L_{d2q3}i_{q3}+L_{d2d3}i_{d3}+L_{d203}i_{03}\end{aligned}\right)i_{q2}+\\&\left(\begin{aligned}&L_{01q2}i_{q1}+L_{01d2}i_{d1}+L_{0102}i_{01}+L_{02q2}i_{q2}+L_{02d2}i_{d2}+\\&L_{0202}i_{02}+L_{02q3}i_{q3}+L_{02d3}i_{d3}+L_{0203}i_{03}\end{aligned}\right)i_{02}\end{aligned}\right] \tag{3.150}$$

$$+\frac{3}{2}\frac{P}{2}\left(\lambda_{pmq2}i_{d2}+\lambda_{pmd2}i_{q2}+\lambda_{pm02}i_{02}\right)$$

Similarly, the co-energy of the machine 3 also can be presented as function of the stator currents and the flux linkages as:

$$W_{co3}=\frac{1}{2}i_{abc3}{}^{t}L_{ss}i_{abc1,2,3}+i_{s1}{}^{t}\lambda_{pm\_3} \tag{3.151}$$

From the torque co-energy equation, the electromagnetic torque can be derived as:

$$T_{e3}=\frac{\partial W_{co3}}{\partial\theta_{rm}} \tag{3.152}$$

The equation (3.152) is equal to the below one.

$$T_{e3}=\frac{1}{2}i_{abc3}{}^{t}\frac{\partial L_{ss}}{\partial\theta_{rm}}i_{abc1,2,3}+i_{abc3}{}^{t}\frac{\partial\lambda_{pm\_3}}{\partial\theta_{rm}} \tag{3.153}$$

Again the mechanical angle of the equation (3.153) needs to be converted to the electrical angle as below:

$$\theta_r = \frac{P}{2}\theta_{rm} \tag{3.154}$$

Therefore, the torque equation is given by:

$$T_{e3} = \frac{P}{2}\frac{1}{2} i_{abc3}{}^{t} \frac{\partial L_{ss}}{\partial \theta_r} i_{abc1,2,3} + \frac{P}{2} i_{s3}{}^{t} \frac{\partial \lambda_{pm\_3}}{\partial \theta_r} \tag{3.155}$$

Substituting the stator currents with their corresponding values in rotor reference frame results in:

$$T_{e3} = \frac{3}{2}\frac{P}{2}(i_{qd3})^{t}\, T(\theta_r)\frac{\partial L_{ss}}{\partial \theta_r} T(\theta_r)^{-1} i_{qd} + \frac{3}{2}\frac{P}{2}(i_{qd3})^{t}\, T(\theta_r)\frac{\partial \lambda_{pm\_3}}{\partial \theta_r} \tag{3.156}$$

The equation can be rewritten as:

$$T_{e3} = \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{q3} \\ i_{o3} \end{bmatrix}^{t} L_{qdo} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{q3} \\ i_{o3} \end{bmatrix}^{t} \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix} \tag{3.157}$$

Replacing the qdo inductances matrix into the equation (3.157) results in:

$$T_{e3} =$$

$$\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix}^t \begin{pmatrix} L_{q11} & L_{q1d1} & L_{q101} & L_{q1q2} & L_{q1d2} & L_{q102} & L_{q1q3} & L_{q1d3} & L_{q103} \\ L_{d1q1} & L_{d11} & L_{d101} & L_{d1q2} & L_{d1d2} & L_{d102} & L_{d1q3} & L_{d1d3} & L_{d103} \\ L_{01q1} & L_{01d1} & L_{0101} & L_{01q2} & L_{01d2} & L_{0102} & L_{01q3} & L_{01d3} & L_{0103} \\ L_{q1q2} & L_{q1d2} & L_{q102} & L_{q22} & L_{q2d2} & L_{q202} & L_{q2q3} & L_{q2d3} & L_{q203} \\ L_{d1q2} & L_{d1d2} & L_{d102} & L_{d2q2} & L_{d22} & L_{d202} & L_{d2q3} & L_{d2d3} & L_{d203} \\ L_{01q2} & L_{01d2} & L_{0102} & L_{02q2} & L_{02d2} & L_{0202} & L_{02q3} & L_{02d3} & L_{0203} \\ L_{q1q3} & L_{q1d3} & L_{q103} & L_{q2q3} & L_{q2d3} & L_{q203} & L_{q33} & L_{q3d3} & L_{q303} \\ L_{d1q3} & L_{d1d3} & L_{d103} & L_{d2q3} & L_{d2d3} & L_{d203} & L_{d3q3} & L_{d33} & L_{d303} \\ L_{01q3} & L_{01d3} & L_{0103} & L_{02q3} & L_{02d3} & L_{0203} & L_{03q3} & L_{03d3} & L_{0303} \end{pmatrix} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{01} \\ i_{q2} \\ i_{d2} \\ i_{02} \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix} + \tag{3.158}$$

$$\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix}^t \begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix}$$

This equation is equal to:

$$
\begin{aligned}
T_{e3} =& \\
&\frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix}^t
\begin{pmatrix}
L_{q11}i_{q1} + L_{q1d1}i_{d1} + L_{q101}i_{01} + L_{q1q2}i_{q2} + L_{q1d2}i_{d2} + L_{q102}i_{02} + L_{q1q3}i_{q3} + L_{q1d3}i_{d3} + L_{q103}i_{03} \\
L_{d1q1}i_{q1} + L_{d11}i_{d1} + L_{d101}i_{01} + L_{d1q2}i_{q2} + L_{d1d2}i_{d2} + L_{d102}i_{02} + L_{d1q3}i_{q3} + L_{d1d3}i_{d3} + L_{d103}i_{03} \\
L_{01q1}i_{q1} + L_{01d1}i_{d1} + L_{0101}i_{01} + L_{01q2}i_{q2} + L_{01d2}i_{d2} + L_{0102}i_{02} + L_{01q3}i_{q3} + L_{01d3}i_{d3} + L_{0103}i_{03} \\
L_{q1q2}i_{q1} + L_{q1d2}i_{d1} + L_{q102}i_{01} + L_{q22}i_{q2} + L_{q2d2}i_{d2} + L_{q202}i_{02} + L_{q2q3}i_{q3} + L_{q2d3}i_{d3} + L_{q203}i_{03} \\
L_{d1q2}i_{q1} + L_{d1d2}i_{d1} + L_{d102}i_{01} + L_{d2q2}i_{q2} + L_{d22}i_{d2} + L_{d202}i_{02} + L_{d2q3}i_{q3} + L_{d2d3}i_{d3} + L_{d203}i_{03} \\
L_{01q2}i_{q1} + L_{01d2}i_{d1} + L_{0102}i_{01} + L_{02q2}i_{q2} + L_{02d2}i_{d2} + L_{0202}i_{02} + L_{02q3}i_{q3} + L_{02d3}i_{d3} + L_{0203}i_{03} \\
L_{q1q3}i_{q1} + L_{q1d3}i_{d1} + L_{q103}i_{01} + L_{q2q3}i_{q2} + L_{q2d3}i_{d2} + L_{q203}i_{02} + L_{q3q3}i_{q3} + L_{q3d3}i_{d3} + L_{q303}i_{03} \\
L_{d1q3}i_{q1} + L_{d1d3}i_{d1} + L_{d103}i_{01} + L_{d2q3}i_{q2} + L_{d2d3}i_{d2} + L_{d203}i_{02} + L_{d3q3}i_{q3} + L_{d3d3}i_{d3} + L_{d303}i_{03} \\
L_{01q3}i_{q1} + L_{01d3}i_{d1} + L_{0103}i_{01} + L_{02q3}i_{q2} + L_{02d2}i_{d2} + L_{0203}i_{02} + L_{03q3}i_{q3} + L_{03d3}i_{d3} + L_{0303}i_{03}
\end{pmatrix} \\
&+ \frac{3}{2}\frac{P}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i_{q3} \\ i_{d3} \\ i_{03} \end{bmatrix}^t
\begin{bmatrix} \lambda_{pmd1} \\ \lambda_{pmq1} \\ \lambda_{pm01} \\ \lambda_{pmd2} \\ \lambda_{pmq2} \\ \lambda_{pm02} \\ \lambda_{pmd3} \\ \lambda_{pmq3} \\ \lambda_{pm03} \end{bmatrix}
\end{aligned}
\tag{3.159}
$$

The non-zero terms of the above equation are:

$$T_{e3} = \frac{3}{2}\frac{P}{2}\left[\begin{matrix}\left(\begin{matrix}L_{q1q3}i_{q1} + L_{q1d3}i_{d1} + L_{q103}i_{01} + L_{q2q3}i_{q2} + L_{q2d3}i_{d2} \\ + L_{q203}i_{02} + L_{q3q3}i_{q3} + L_{q3d3}i_{d3} + L_{q303}i_{03}\end{matrix}\right)i_{d3} - \\ \left(\begin{matrix}L_{d1q3}i_{q1} + L_{d1d3}i_{d1} + L_{d103}i_{01} + L_{d2q3}i_{q2} + L_{d2d3}i_{d2} \\ + L_{d203}i_{02} + L_{d3q3}i_{q3} + L_{d3d3}i_{d3} + L_{d303}i_{03}\end{matrix}\right)i_{q3} + \\ \left(\begin{matrix}L_{01q3}i_{q1} + L_{01d3}i_{d1} + L_{0103}i_{01} + L_{02q3}i_{q2} + L_{02d2}i_{d2} \\ + L_{0203}i_{02} + L_{03q3}i_{q3} + L_{03d3}i_{d3} + L_{0303}i_{03}\end{matrix}\right)i_{03}\end{matrix}\right] + \frac{3}{2}\frac{P}{2}\left(\lambda_{pmq3}i_{d3} + \lambda_{pmd2}i_{q3} + \lambda_{pm02}i_{03}\right) \tag{3.160}$$

The dynamic governing rotor speed can also be derived using the electromagnetic and load torque. In the below equation *'P'* is the pole pairs of the machine, 'ω_r' is the rotor speed, 'B' is the friction coefficient and 'T_L' is the mechanical load torque applied to the machine.

$$T_e = T_{e1} + T_{e2} + T_{e3} = J\left(\frac{2}{P}\right)p\omega_r + T_L + B\omega_r \tag{3.161}$$

3.8.2 Generating the Parameters of the Asymmetrical Triple Star IPM

After deriving the model, the next step is to determine the machine parameters to be used for the modelling of the machine. The stator inductances can be derived from the clock diagram of the machine. The clock diagram of the machine is shown in the Figure 3.103. As it can be seen each machine has four poles with full pitch, double layer and concentrated windings. The machine has 36 slots each slot covers 20 degrees of the stator circumferential and also each machine is shifted by 20 degrees from the adjacent machine. In this diagram the machine 1 is shown in blue, machine 2 is shown in red and machine 3 is shown in green color. The turn function of each machine can be generated from the clock diagram. Figures 3.104 to 3.105 show the turn function for each of the machines.

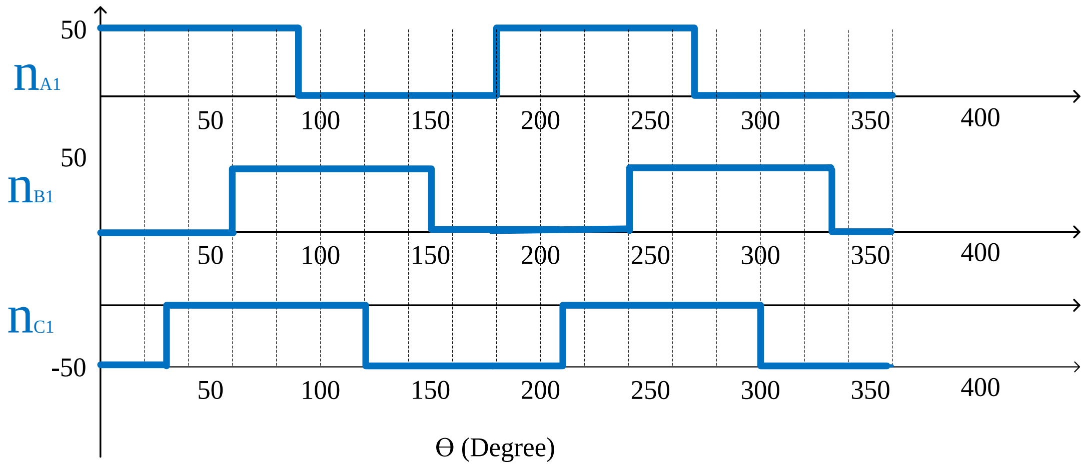


Figure 3.104: The turn functions of the machine 1 phases.

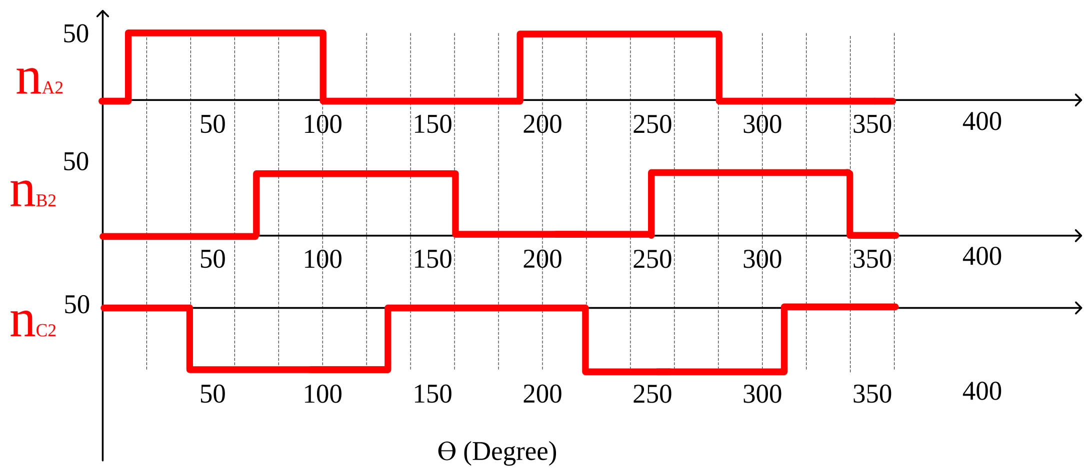


Figure 3.105: The turn functions of the machine 2 phases.

The rotor of the machine is the same as the rotor of the single star machine shown in the Figure 3.5 also the airgap function is shown in the Figure 3.6. Again using the wave forms of the turn functions, airgap function and the equation (3.162) the winding function of the phase stator phases are generated as a function of rotor and circumferential angle of the stator. Figure 3.107 shows the winding function of the phase ‘a’ of the machine 1 [83].

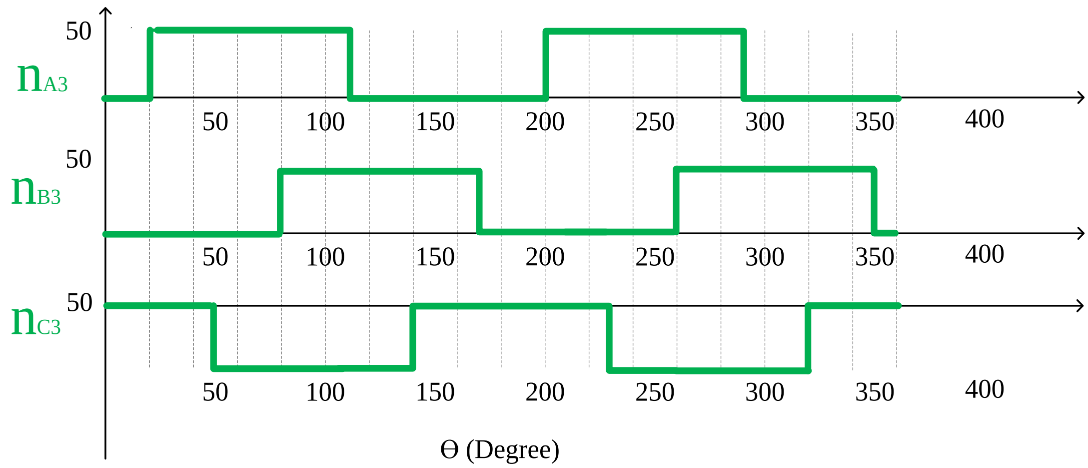


Figure 3.106: The turn functions of the machine 3 phases.

$$N_w(\theta) = n_w(\theta) - \frac{\int_0^{2\pi} \frac{n_w(\theta)}{g(\theta,\theta_r)} d\theta}{\int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} d\theta} \tag{3.162}$$

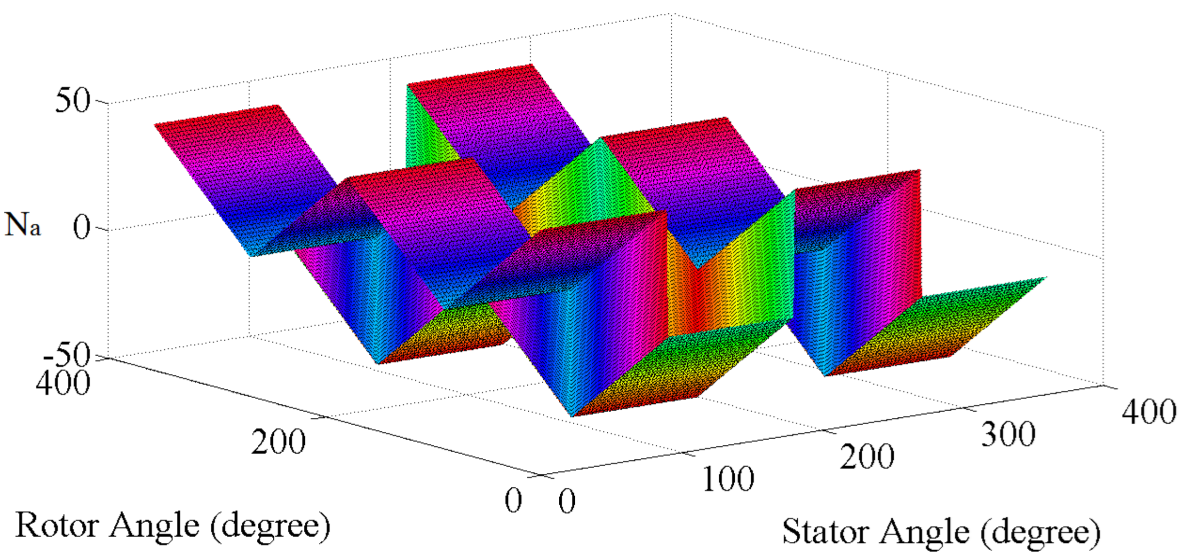


Figure 3.107: The winding function phase ‘a’ of the machine 1.

The winding functions of the rest of the phases have the same form except they are shifted by 20 degree of the stator angle. Using the generated winding functions and equation (3.163) the self and mutual inductances of the machine phases can be calculated. The Figures 3.108 to 3.113 show the mutual and self-inductances of the machine corresponding to each stator phase [83].

$$L_{jk} = \mu_o r l \int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} n_j(\theta) N_k(\theta) d\theta \tag{3.163}$$

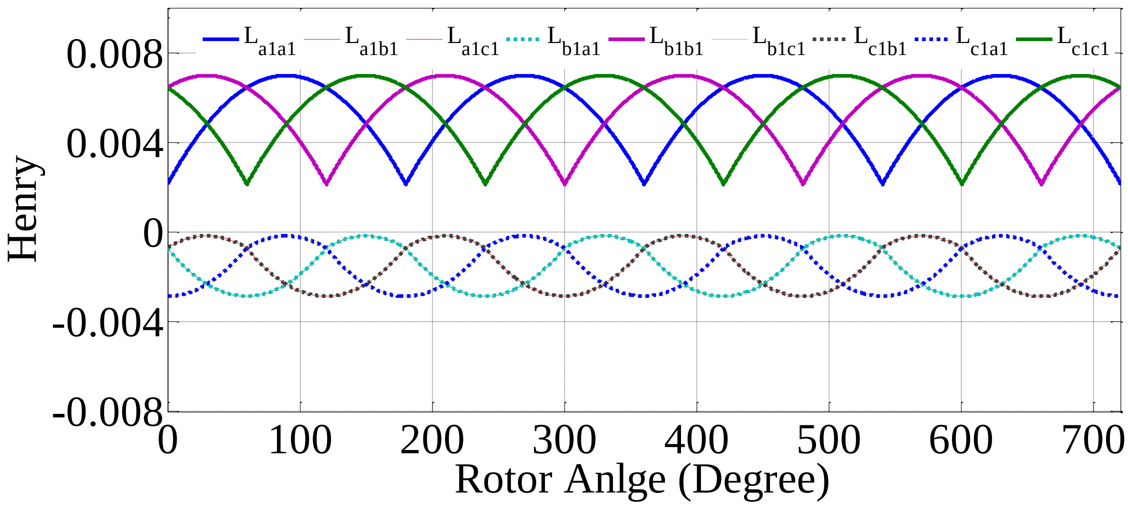


Figure 3.108: The self and mutual inductances corresponding to phases of machine 1.

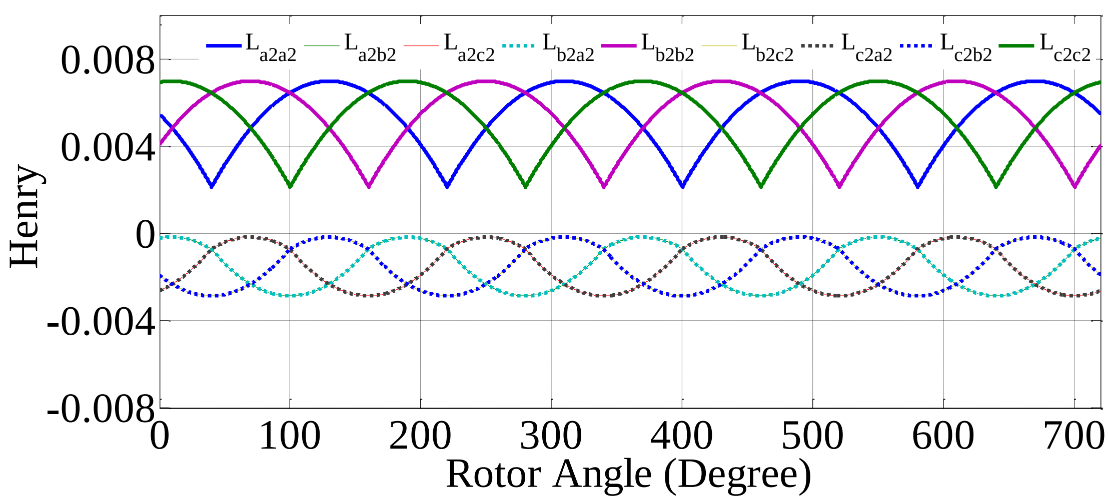


Figure 3.109: The self and mutual inductances corresponding to phases of machine 2.

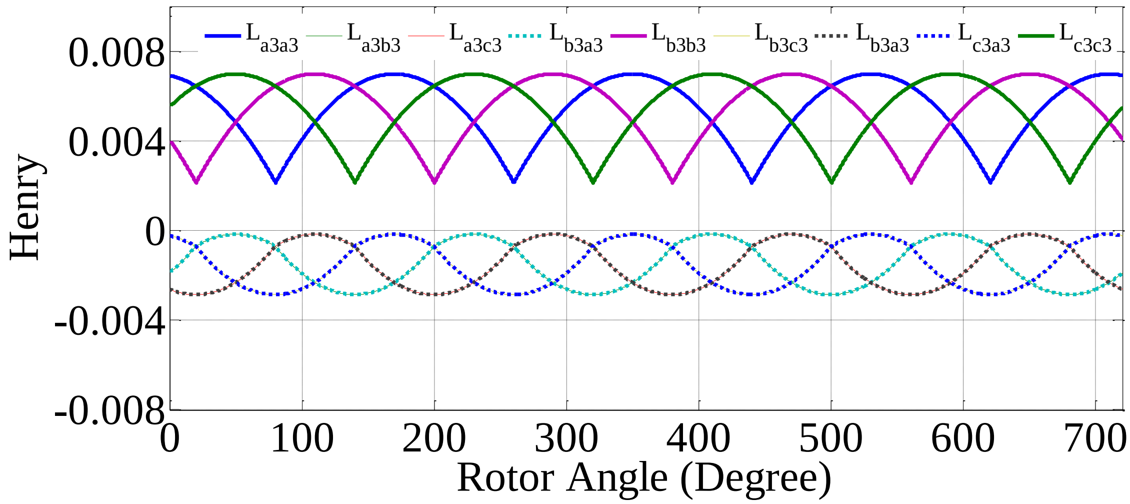


Figure 3.110: The self and mutual inductances corresponding to phases of machine 3.

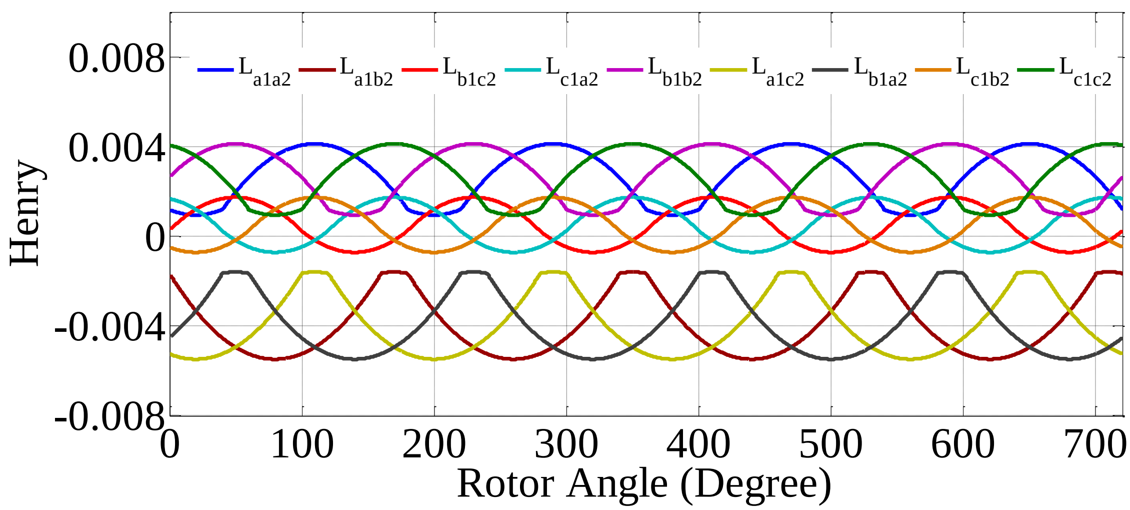


Figure 3.111: The mutual inductances between machines 1 and 2.

Again the inductances are functions of the angle θ_r. Stator self-inductances are maximum when the rotor q-axis is aligned with the phase, and mutual inductances are maximum when the rotor q-axis is in the midway between two phases. Now the inductances can be arranged in the matrix form as presented in equation (3.164) and using the equations (3.123), they can be transformed to the rotor reference frame to get the q and d inductances of each machine.

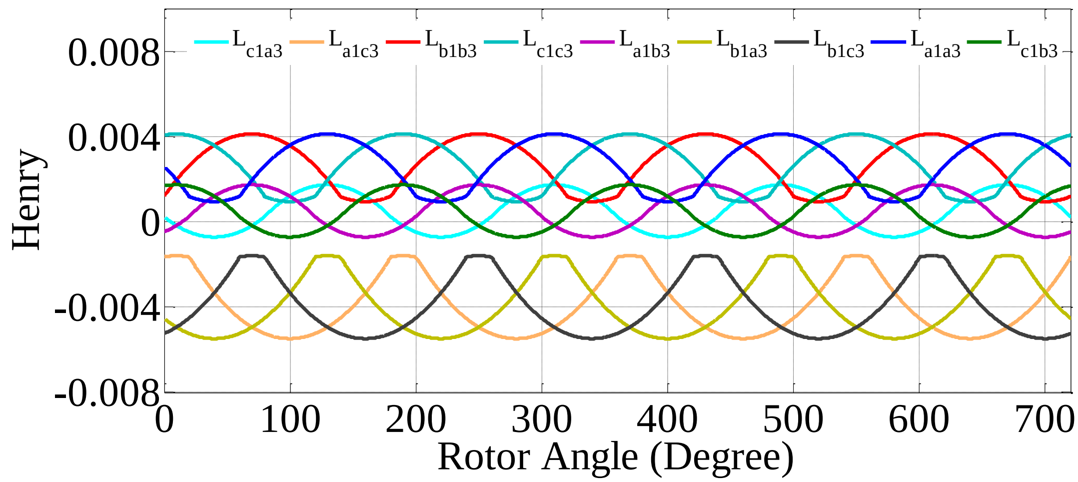


Figure 3.112: The mutual inductances between machines 1 and 3.

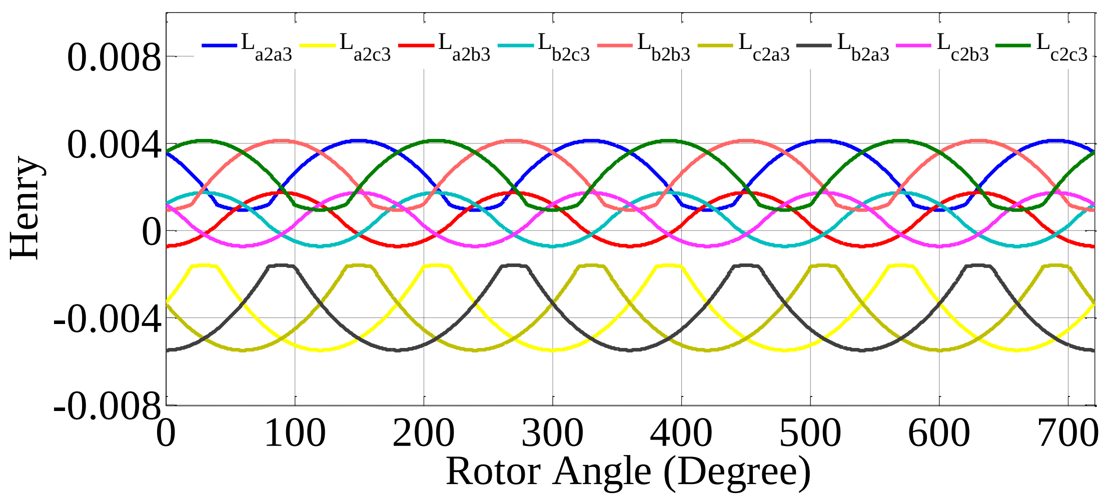


Figure 3.113: The mutual inductances between machines 2 and 3.

$$L_{ss} = \begin{pmatrix} L_{ls}+L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} & L_{a1a3} & L_{a1b3} & L_{a1c3} \\ L_{b1a1} & L_{ls}+L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} & L_{b1a3} & L_{b1b3} & L_{b1c3} \\ L_{c1a1} & L_{c1b1} & L_{ls}+L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} & L_{c1a3} & L_{c1b3} & L_{c1c3} \\ L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{ls}+L_{a2a2} & L_{a2b2} & L_{a2c2} & L_{a2a3} & L_{a2b3} & L_{a2c3} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{ls}+L_{b2b2} & L_{b2c2} & L_{b2a3} & L_{b2b3} & L_{b2c3} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{ls}+L_{c2c2} & L_{c2a3} & L_{c2b3} & L_{c2c3} \\ L_{a3a1} & L_{a3b1} & L_{a3c1} & L_{a3a2} & L_{a3b2} & L_{a3c2} & L_{ls}+L_{a3a3} & L_{a3b3} & L_{a3c3} \\ L_{b3a1} & L_{b3b1} & L_{b3c1} & L_{b3a2} & L_{b3b2} & L_{b3c2} & L_{b3a3} & L_{ls}+L_{b3b3} & L_{b3c3} \\ L_{c3a1} & L_{c3b1} & L_{c3c1} & L_{c3a2} & L_{c3b2} & L_{c3c2} & L_{c3a3} & L_{c3b3} & L_{ls}+L_{c3c3} \end{pmatrix} \tag{3.164}$$

The generated inductances in rotor reference frame are shown in the Figures 3.114 to 3.119.

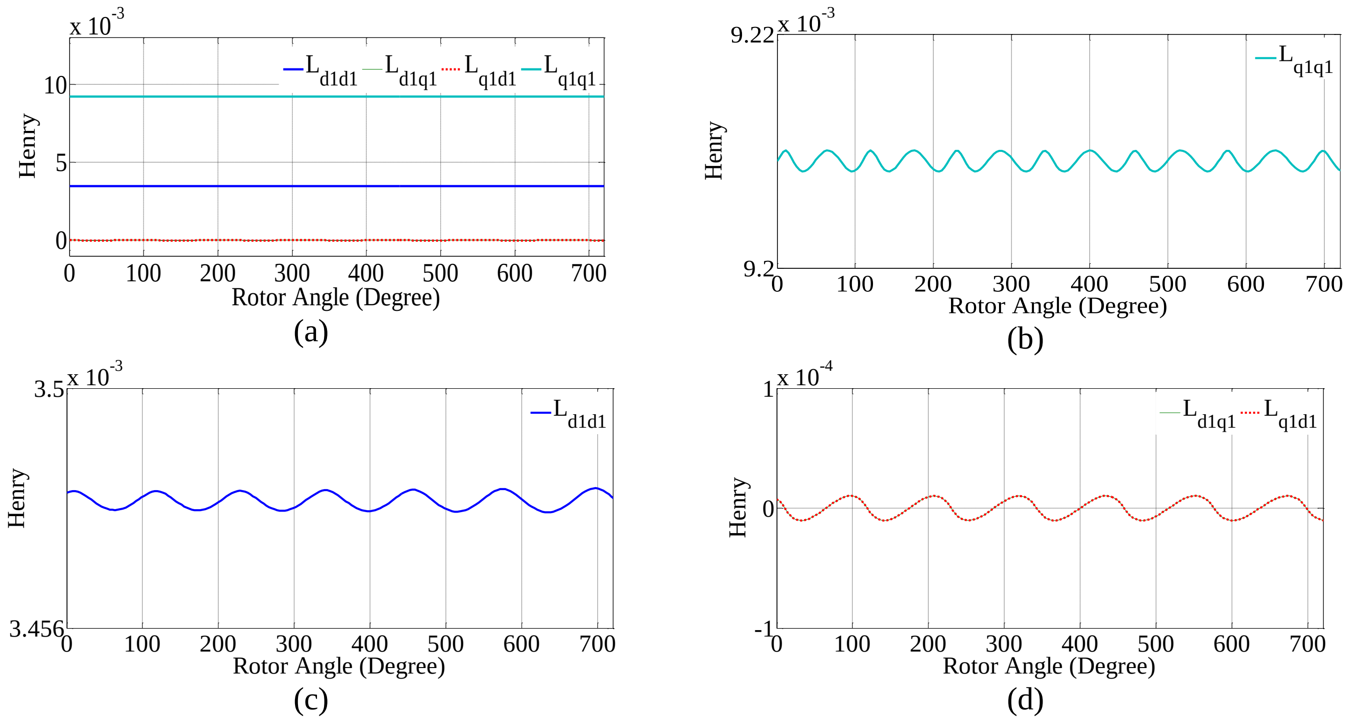


Figure 3.114: (a) The inductances of the machine 1 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

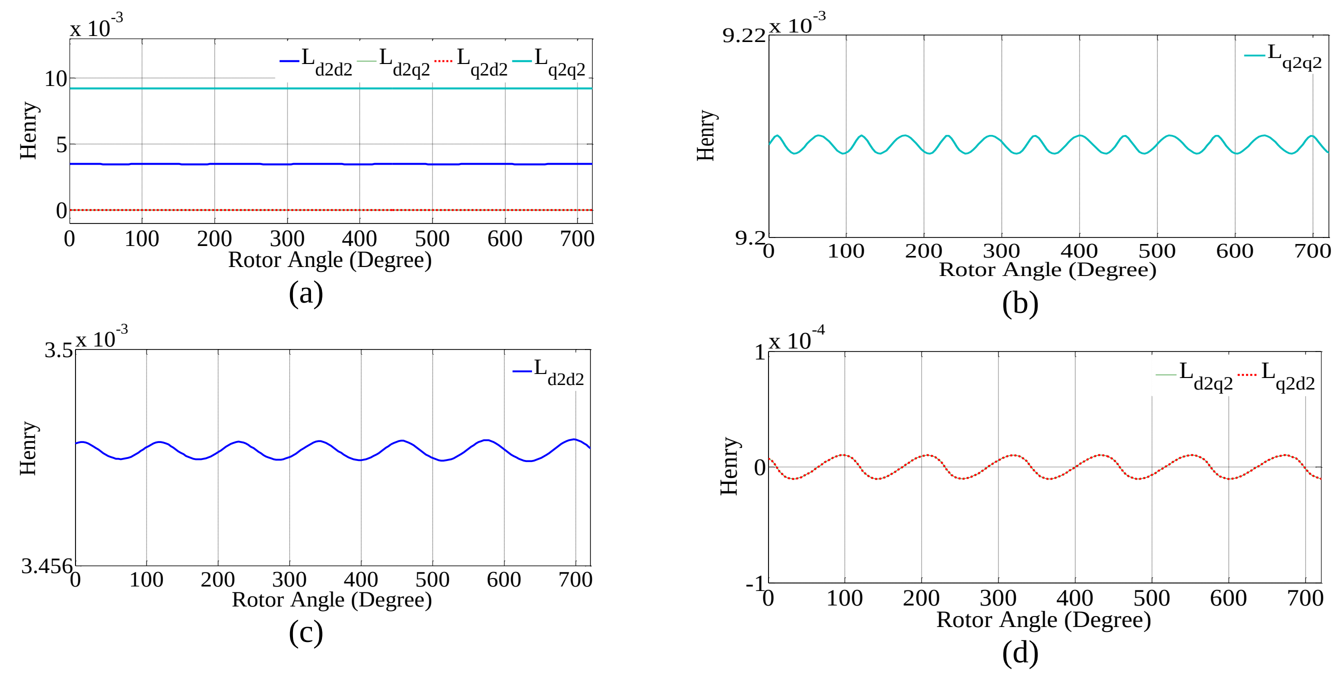


Figure 3.115: (a) The inductances of the machine 2 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

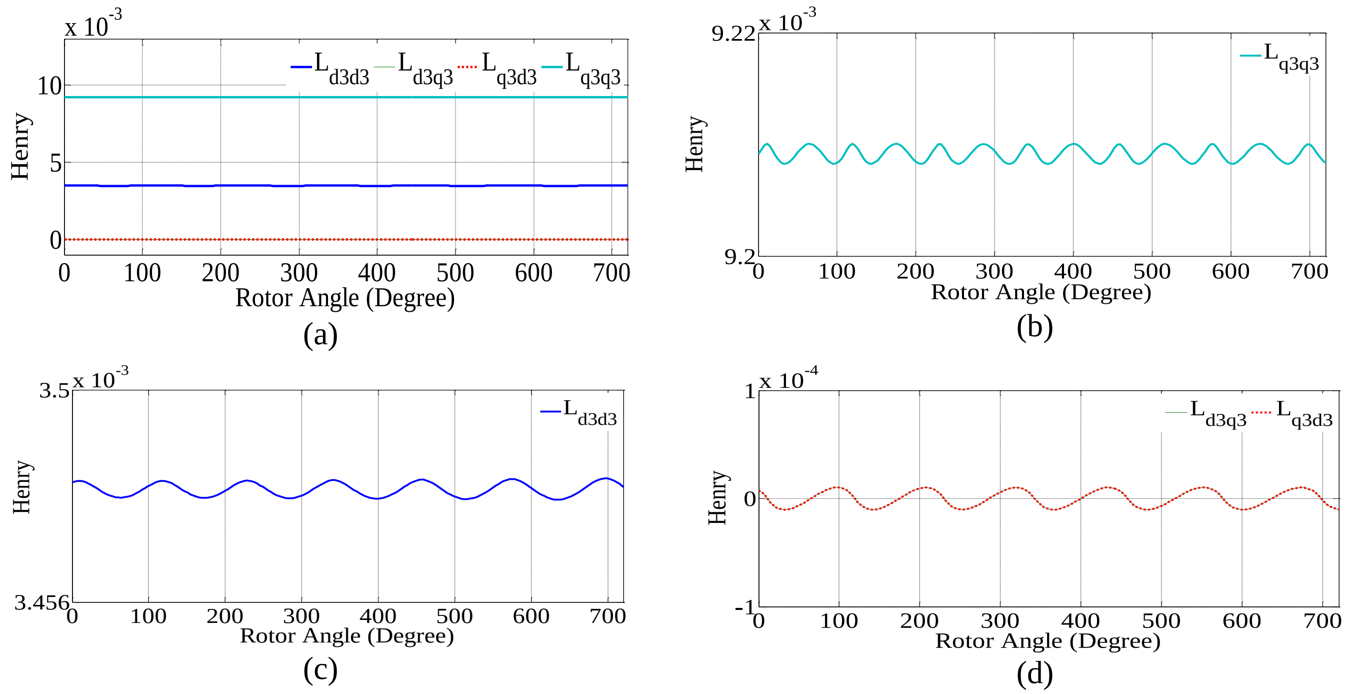


Figure 3.116: (a) The inductances of the machine 3 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

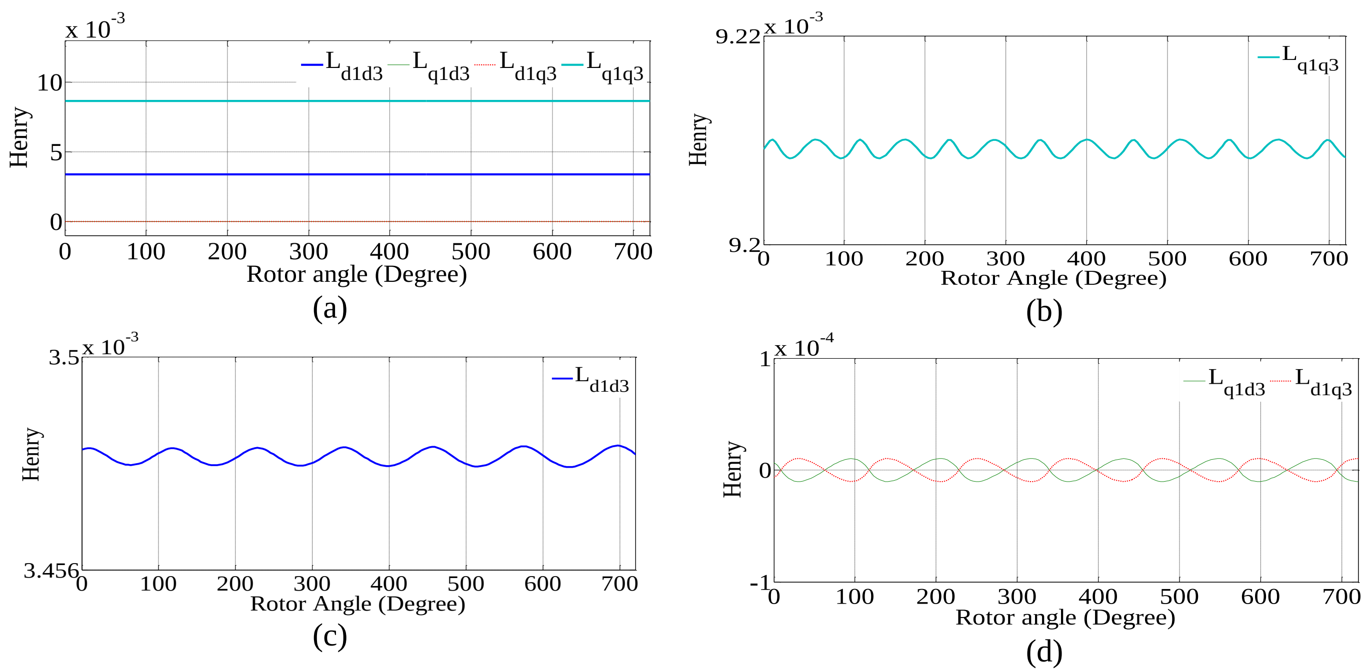


Figure 3.117: (a) The mutual inductances between the machines 1 and 3 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

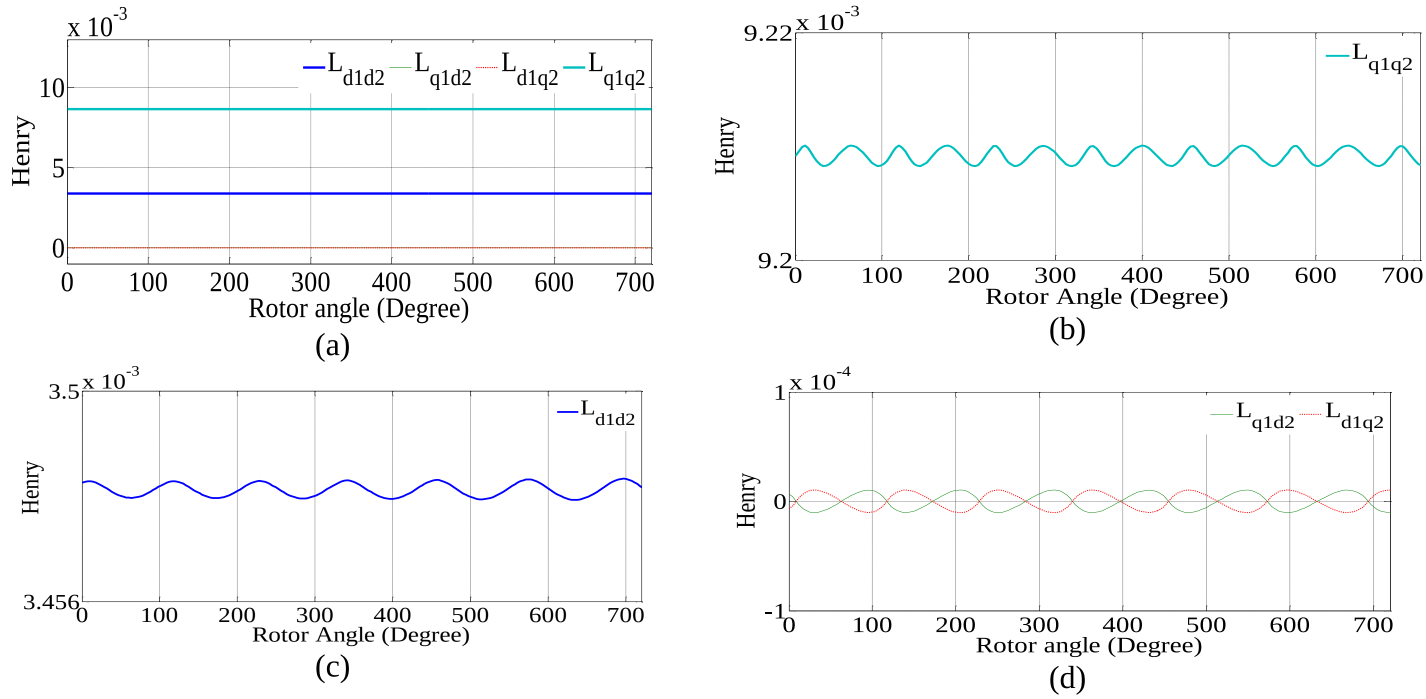


Figure 3.118: (a) The mutual inductances between the machines 1 and 2 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

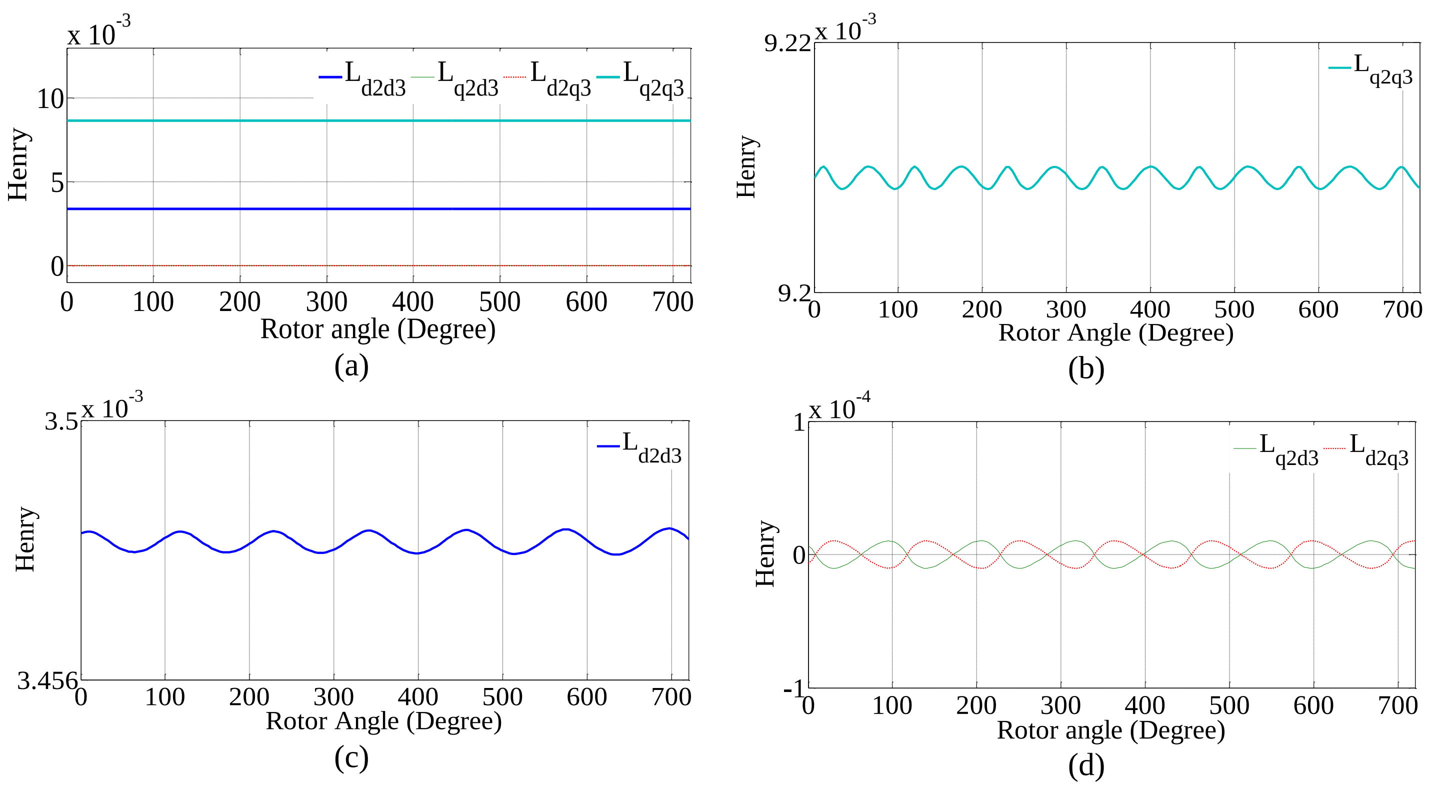


Figure 3.119: (a) The mutual inductances between the machines 2 and 3 in the rotor reference frame, The zoomed view of, (b) The q axis inductance, (c) The d axis inductance, (d) The mutual between q and d axis.

The equation (3.36) is repeated here to show the flux linkage due to the rotor permanent magnets seen from the phase ‘a’ of machine 1. The flux linkages seen from the phase ‘a’ of machines 2 and 3 have the same shape but 20-degree phase shift from each other.

$$B_r(\theta_r) = \begin{cases} B_{\max} & -\frac{90^o - \alpha_p}{2} < |\theta_r| < \frac{90 - \alpha_p}{2}\ ,\ 135^o - \alpha_p < |\theta_r| < 180^o \\ B_{\max} - \frac{2B_{\max}}{\alpha_p}\theta_r & \frac{90^o - \alpha_p}{2} < |\theta_r| < 90^o \\ -B_{\max} & 90^o < |\theta_r| < 135^o - \frac{3\alpha_p}{2} \\ -B_{\max} + \frac{2B_{\max}}{\alpha_p}\theta_r & 135^o - \frac{3\alpha_p}{2} < |\theta_r| < 135^o - \alpha_p \end{cases} \quad (3.165)$$

Now according to equation (3.122) the flux linkage due to the permanent magnets on each phase of the stator can be transformed to the rotor reference frame to get the *q* and *d* axis flux linkages for each of the machines. Figures 3.120 to 3.125 show the flux linkage components for each machine.

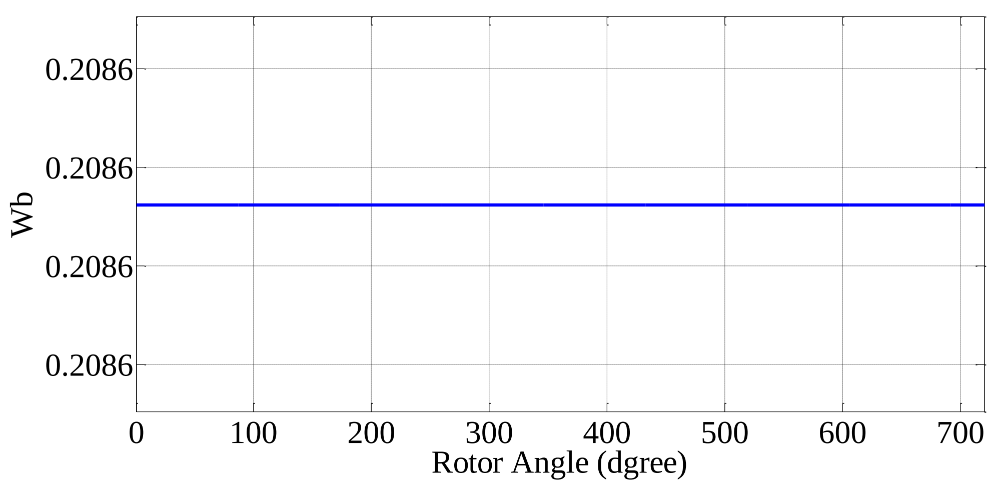


Figure 3.120: The d axis flux linkage due to the rotor permanent magnets corresponding to machine 1.

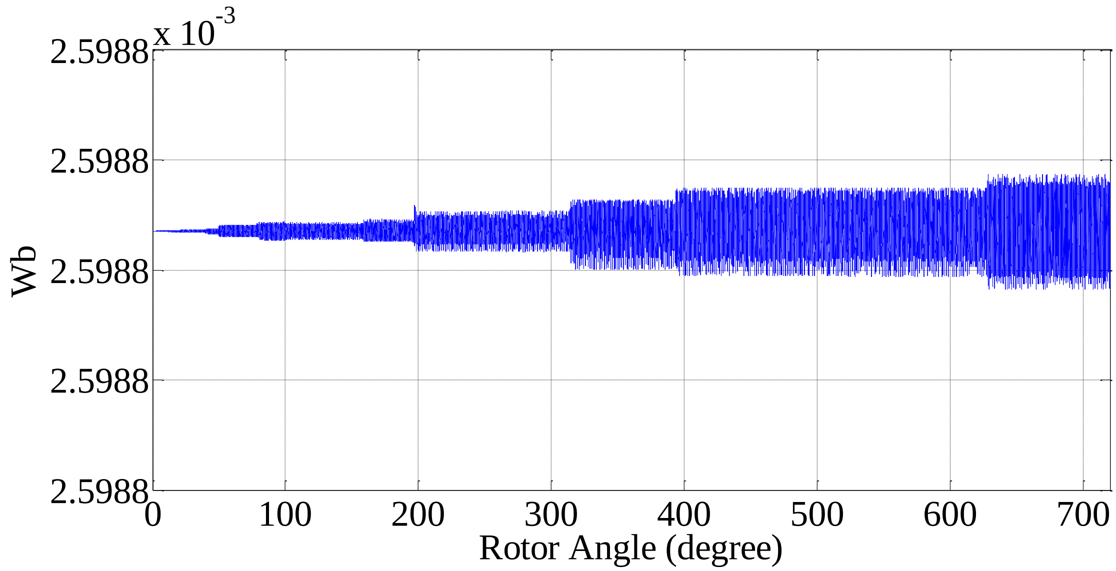


Figure 3.121: The q axis flux linkage due to the rotor permanent magnets corresponding to machine 1.

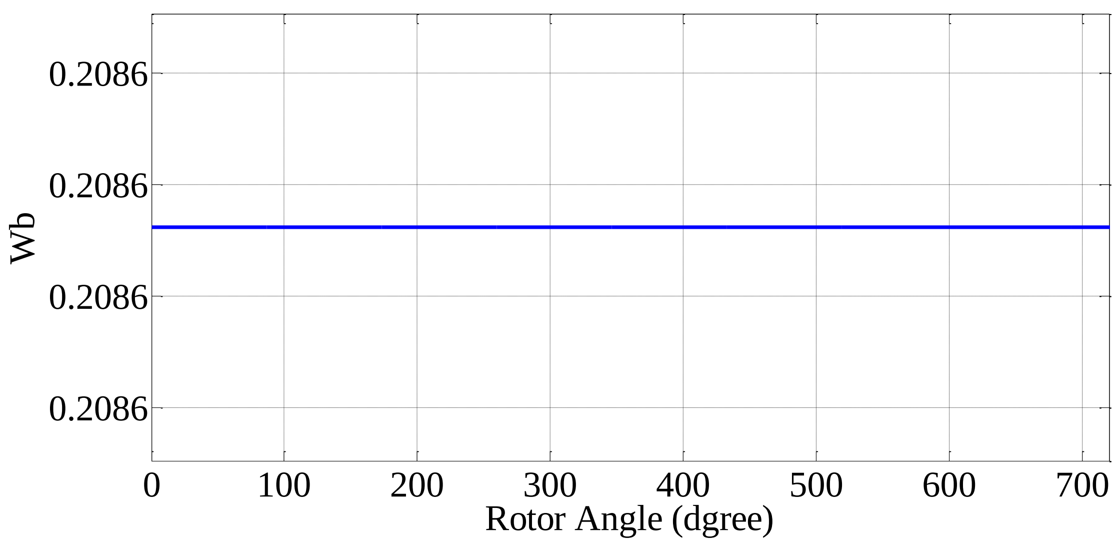


Figure 3.122: The d axis flux linkage due to the rotor permanent magnets corresponding to machine 2.

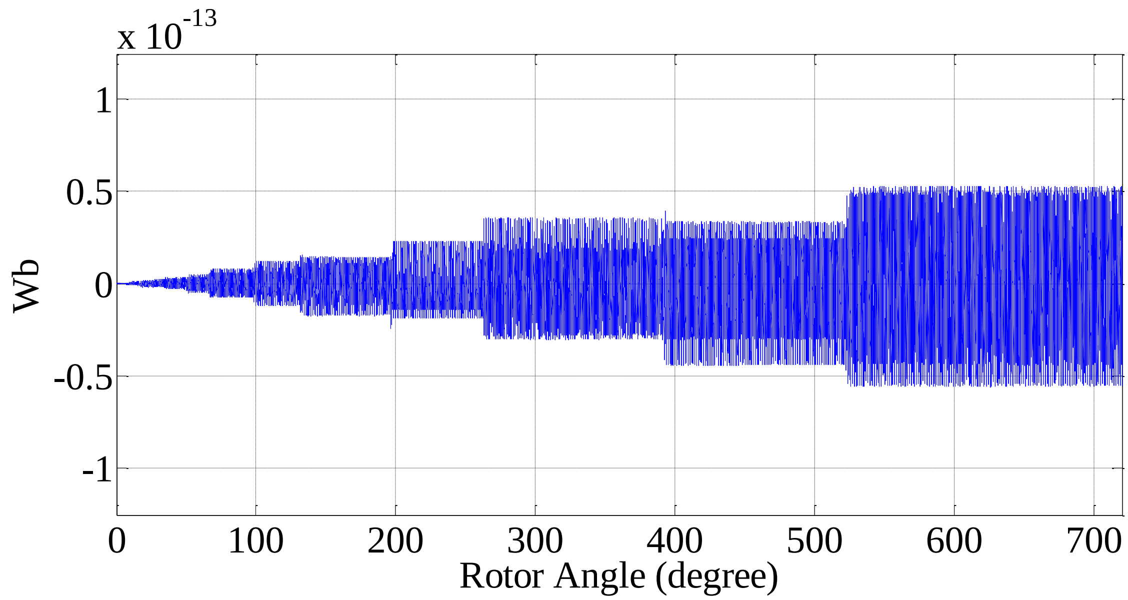


Figure 3.123: The q axis flux linkage due to the rotor permanent magnets corresponding to machine 2.

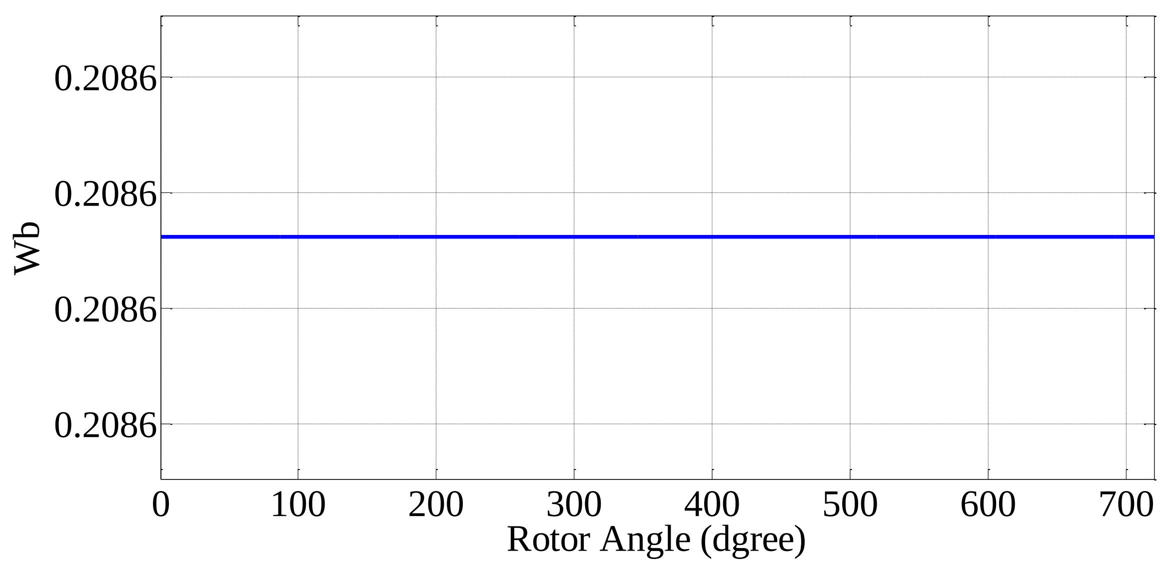


Figure 3.124: The d axis flux linkage due to the rotor permanent magnets corresponding to machine 3.

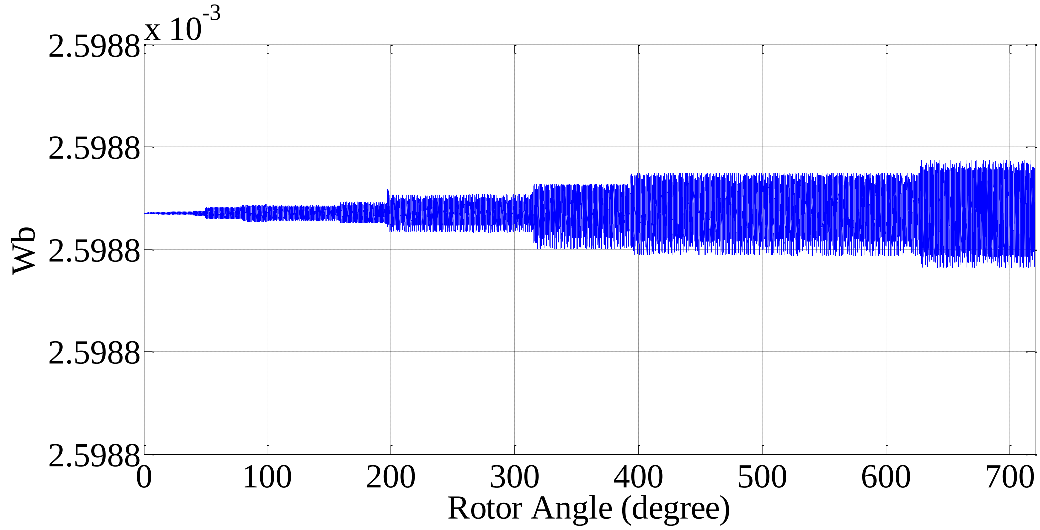


Figure 3.125: The q axis flux linkage due to the rotor permanent magnets corresponding to machine 3.

3.8.3 Simulation of the Asymmetrical Triple Star IPM

The inductances and the permanent magnet flux linkages that are generated in previous section could be used in the machine equations to simulate the machine. Using all generated equations and wave forms for the machine and substituting the machine parameters presented in the Table 3.1, the asymmetrical triple star IPM machine can be simulated using MATLAB/ Simulink. Three sets of 60 (Hz) 110 (Volts) three-phase voltages (as shown in Figure 3.126) are applied to the model while the initial rotor speed is 377 rad/sec. The machine speed passes some transients at the beginning and it goes to the steady state. When the machine is at the steady state 5 N.m load torque is applied to the machine. The simulation results are shown in the following figures. Figure 3.127 shows the rotor speed it can be seen that after initial transients the rotor speed goes to the synchronous speed which is equal to the source frequency.

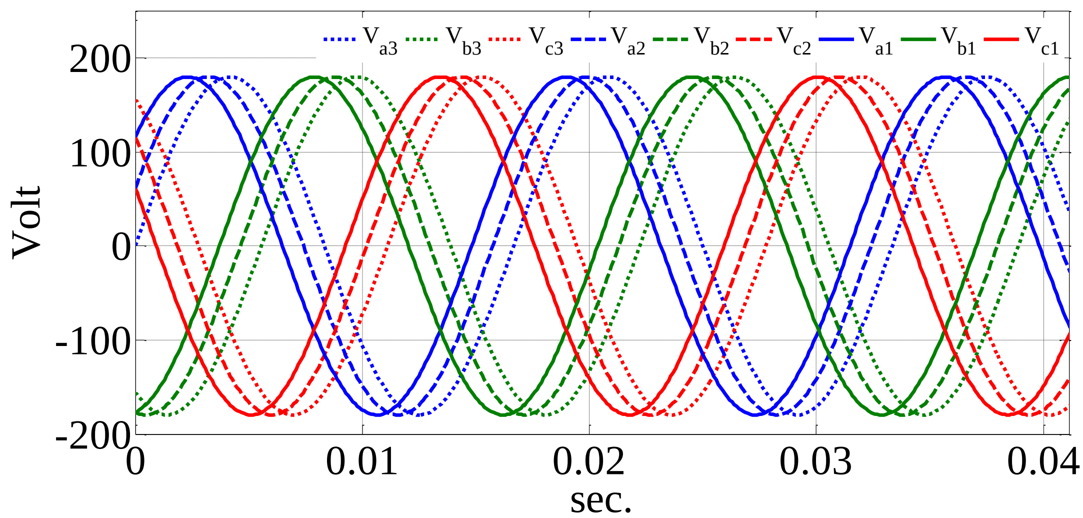


Figure 3.126: The stator phase voltages.

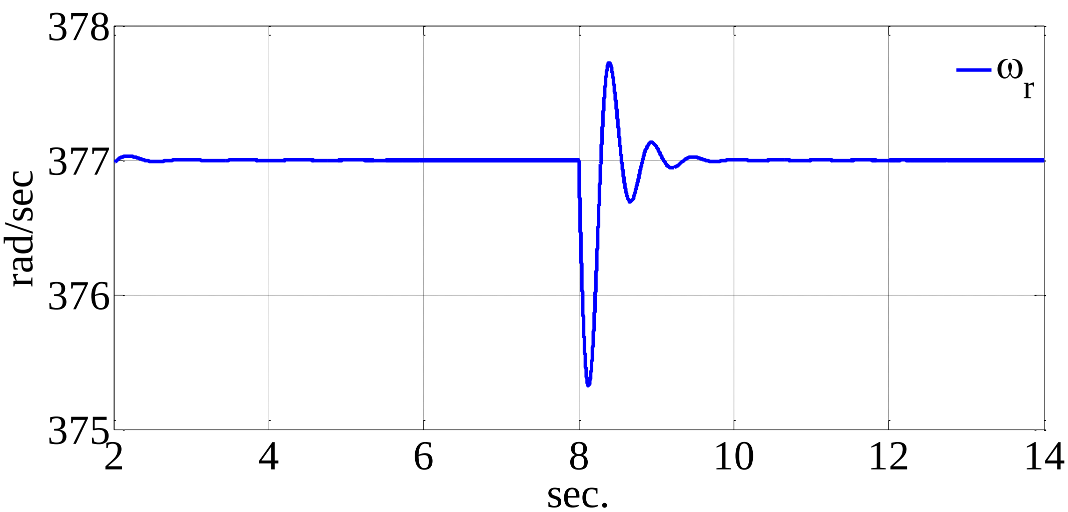


Figure 3.127: The rotor speed.

Figure 3.128 shows the electromagnetic and load torque together. As it can be seen at the start point the machine has the transients and finally it settles down to the steady state. After applying the load, the machine starts generating electromagnetic torque to keep the synchronous speed. Figure 3.129 shows the spectrum of the torque and the zoomed view of that. It can be seen that along with the DC component the components with the higher frequency also exist in the electromagnetic torque. This torque is the generated by three machines and each of them shares a part of that.

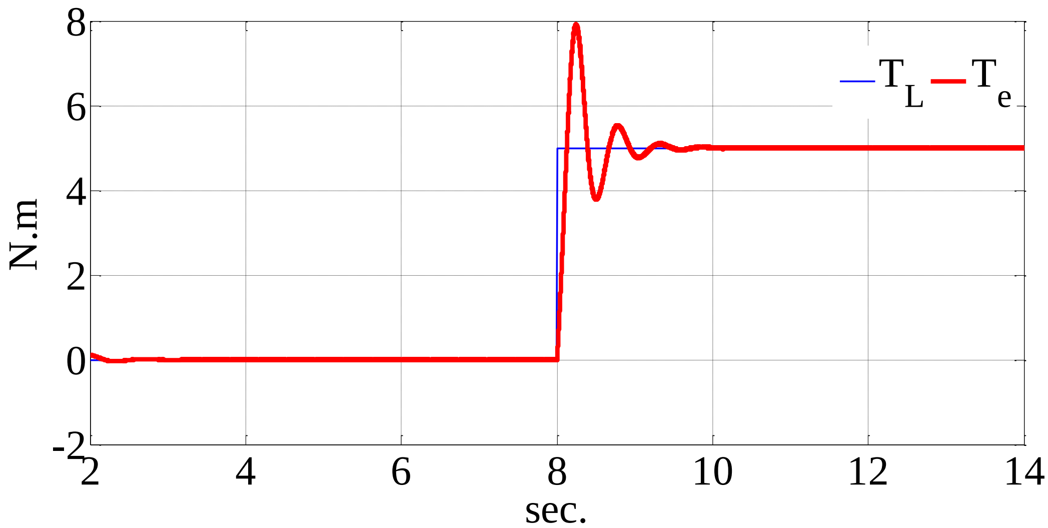


Figure 3.128: The electromagnetic torque generated by all machines.

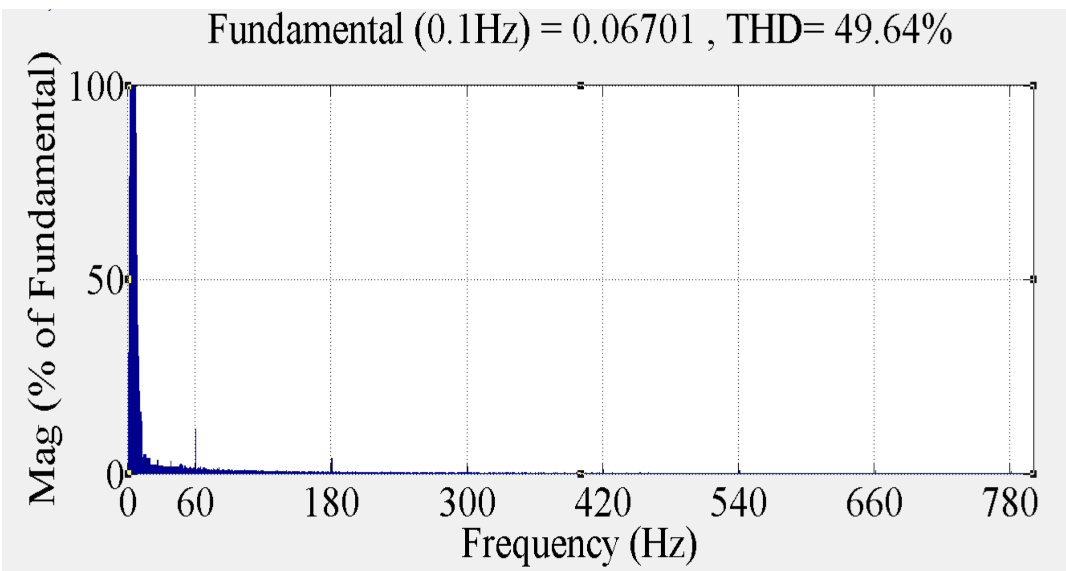
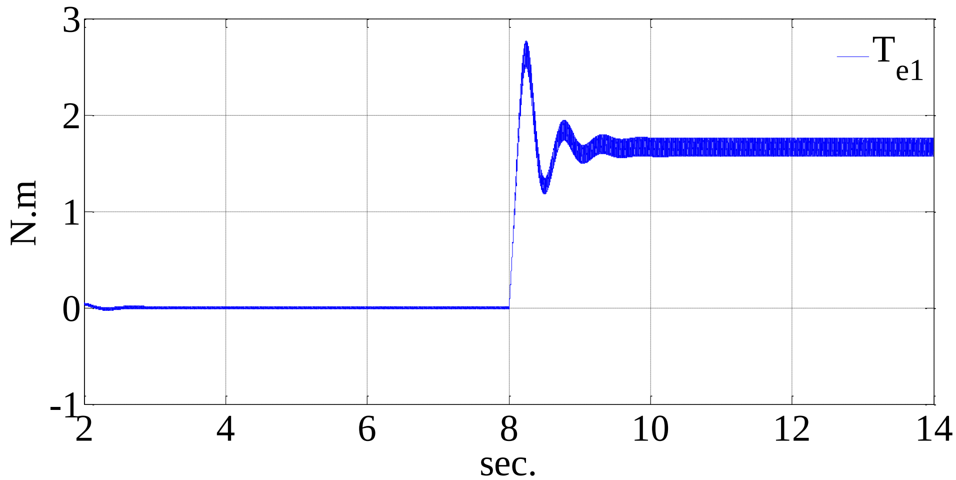
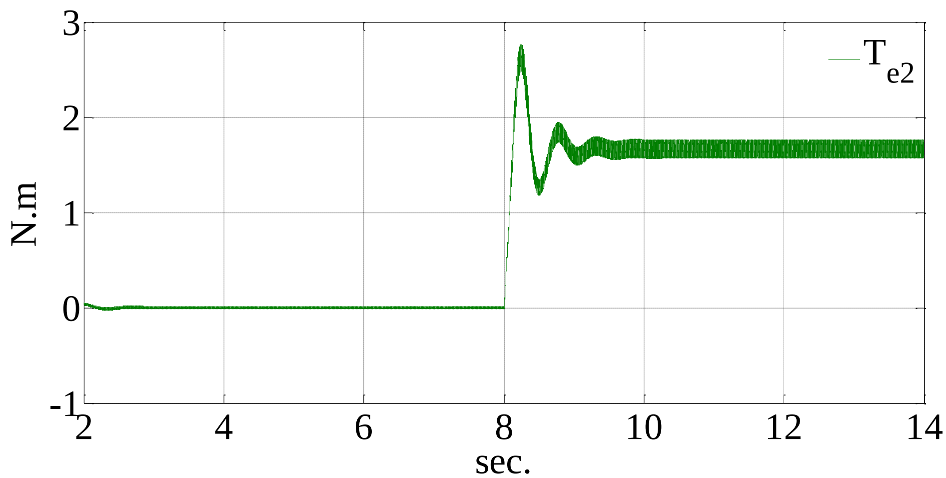


Figure 3.129: The spectrum of the electromagnetic torque.

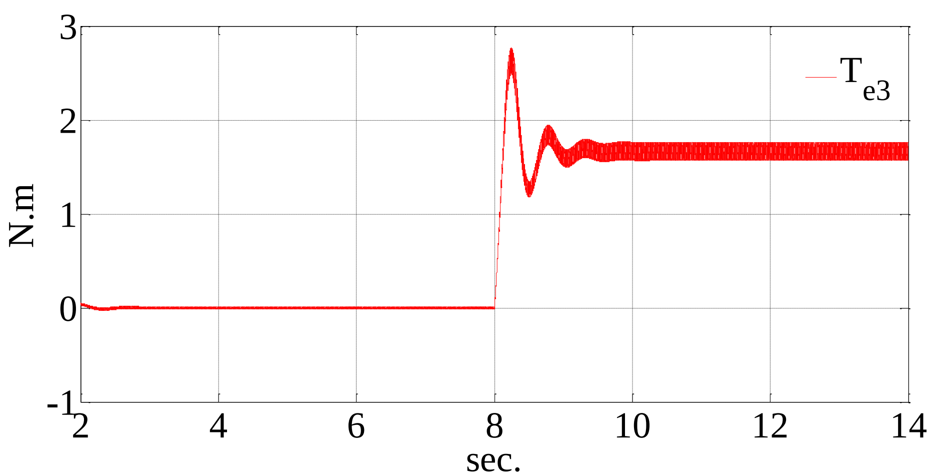
Figure 3.130 shows the electromagnetic torque of each machine separately. And the spectrum of the electromagnetic torque of each machine is shown in the Figure 3.131. The zoomed view of the electromagnetic torques are shown in the Figure 3.132.



(a)

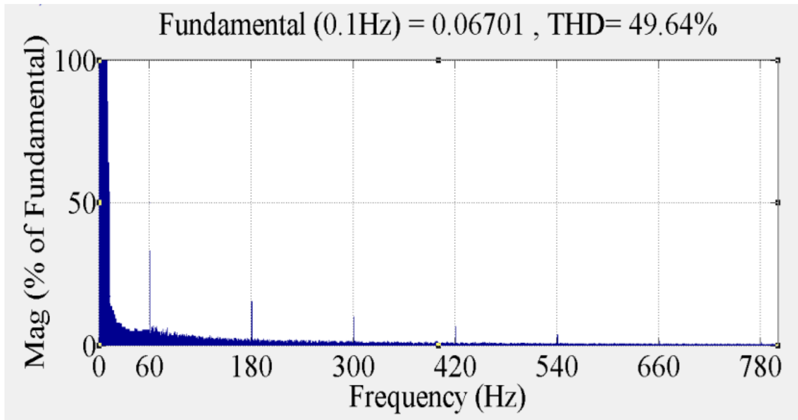


(b)

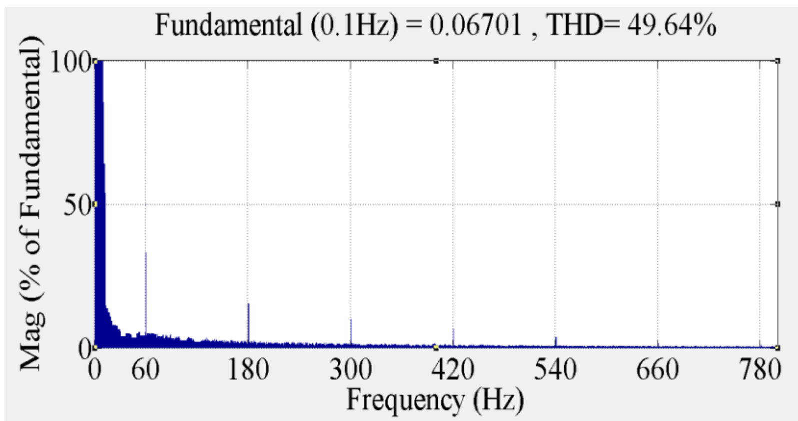


(c)

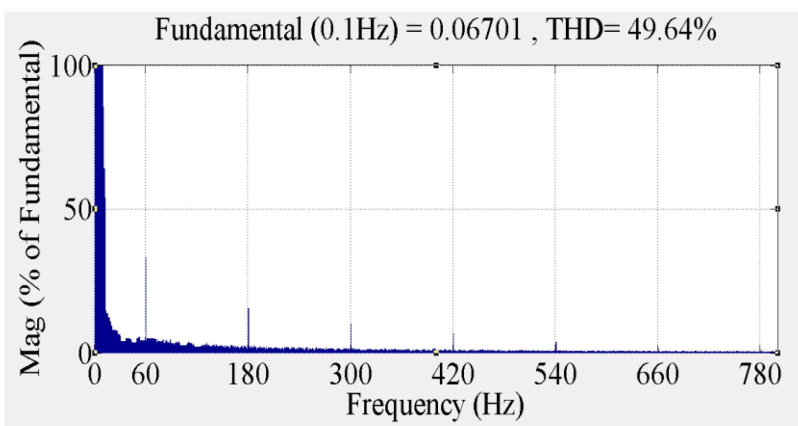
Figure 3.130: The electromagnetic torque generated by each machine, (a) Machine 1, (b) Machine 2, (c) Machine 3.



(a)



(b)



(c)

Figure 3.131: The spectrum of the electromagnetic torque generated by each machine, (a) Machine 1, (b) Machine 1, (c) Machine 3.

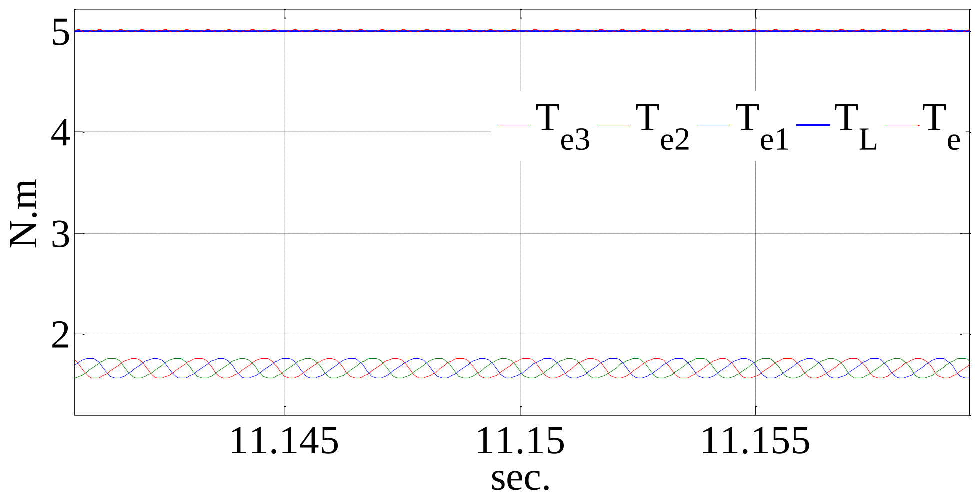


Figure 3.132: The zoomed view of the total electromagnetic torque and electromagnetic torque generated by each machine.

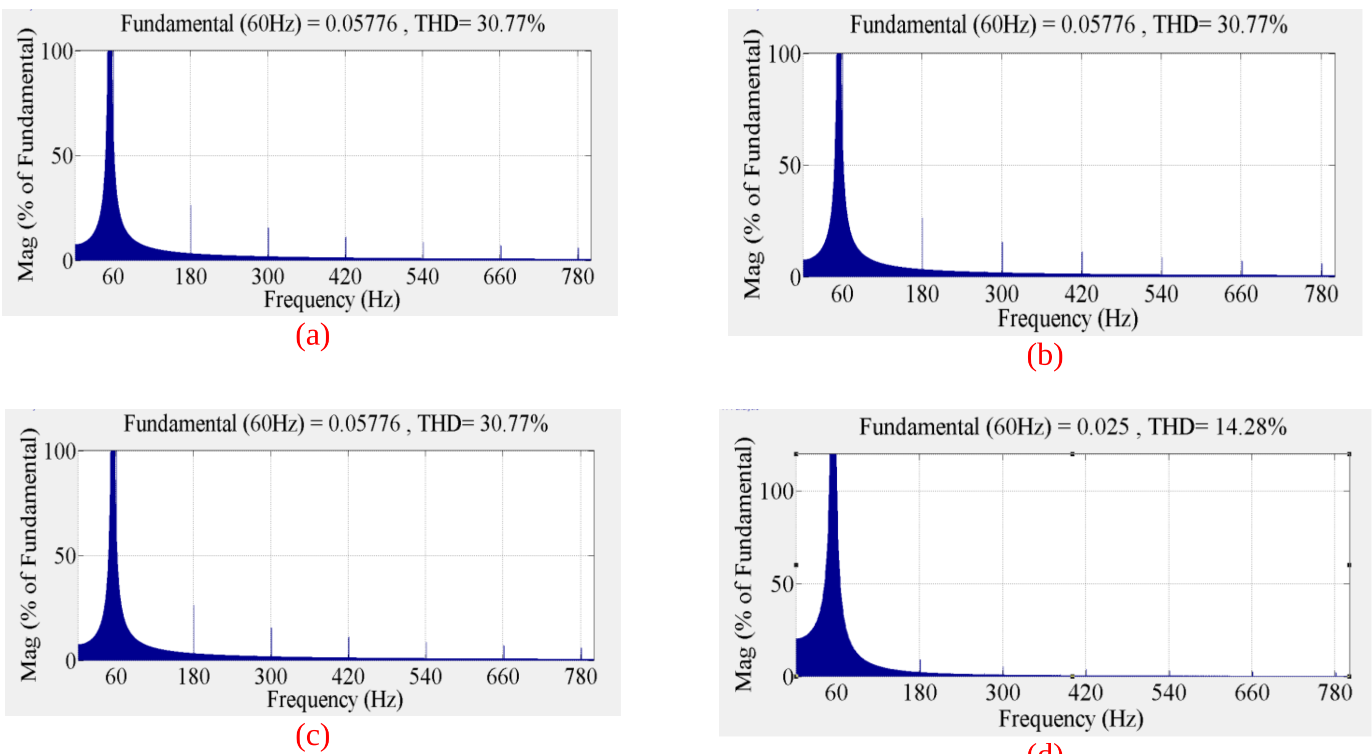


Figure 3.133: The spectrum of the airgap flux linkage for, (a) The machine '1', (b) The machine '2', (c) The machine '3', (d) The total.

The spectrums of the airgap flux linkage for each machine and also the total flux linkage are shown in the Figure 3.133. By comparing the Figures 3.133 (a), (b) and (c) with 3.133 (d), it can be seen that due to the harmonic cancellation the total flux linkage has lower harmonics magnitude compared to each individual machine. The stator currents in natural quantities are shown in the Figure 3.134. The transients can be seen at the starting and load changing moment, when transient is passed and the machine goes to steady state the currents also settle down to their final values. Also the Figures 3.136 to 3.138 show the currents of each machine separately during their steady state along with the spectrum of the phase ‘a’ if each machine.

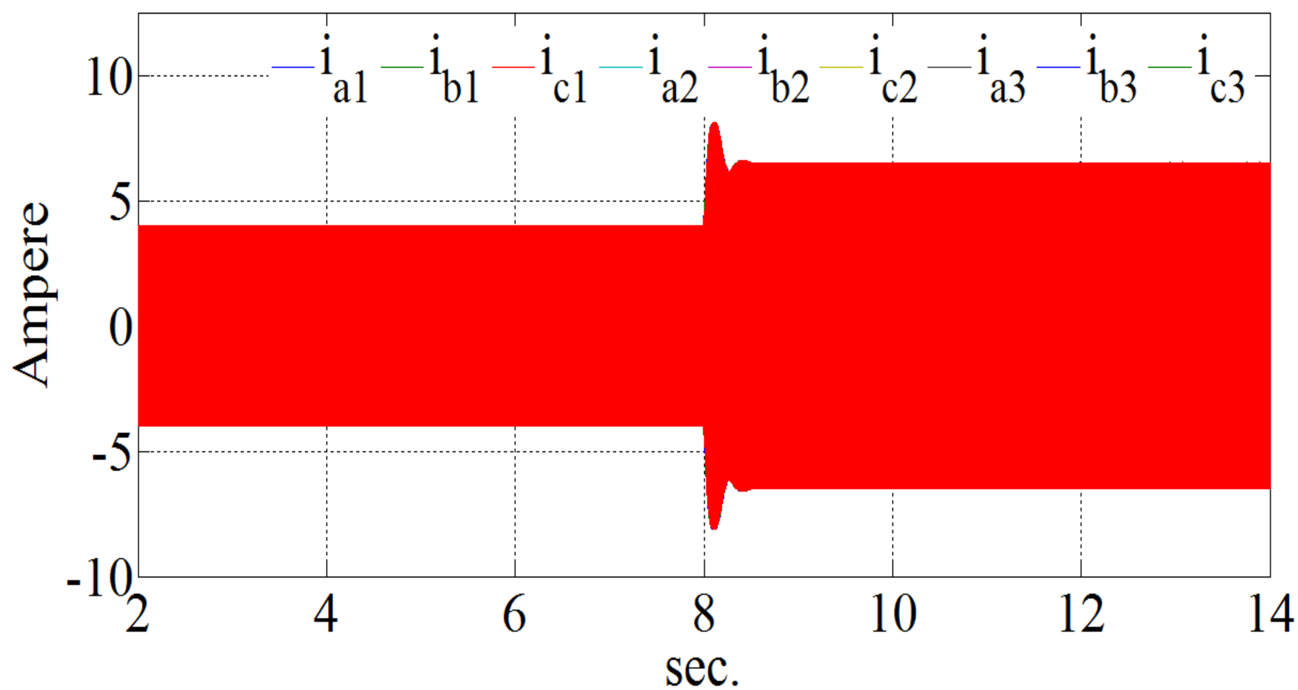
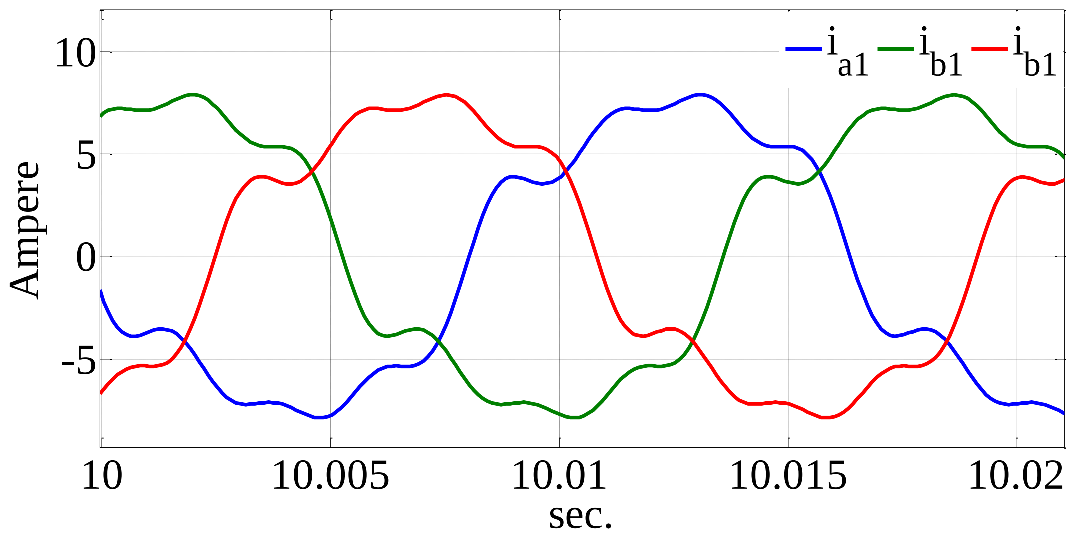
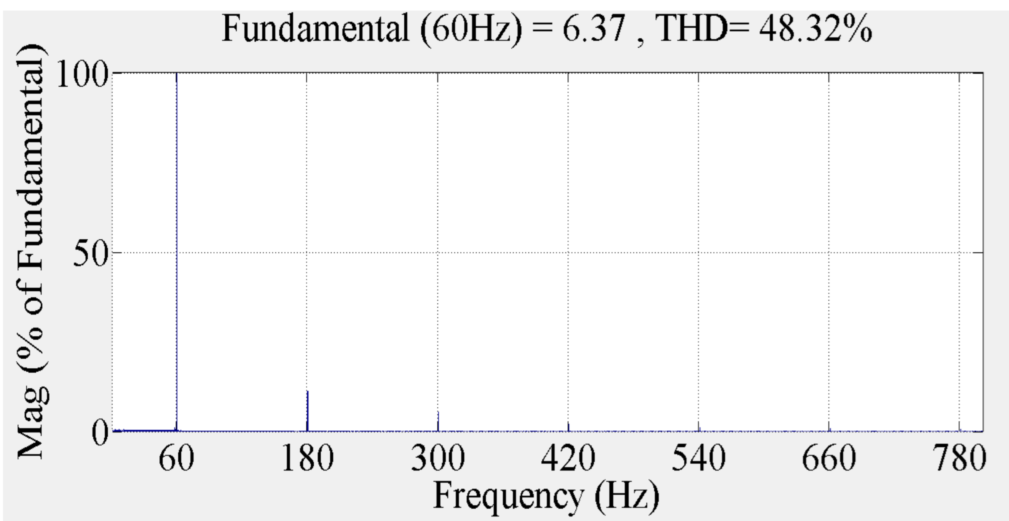


Figure 3.134: The stator currents.

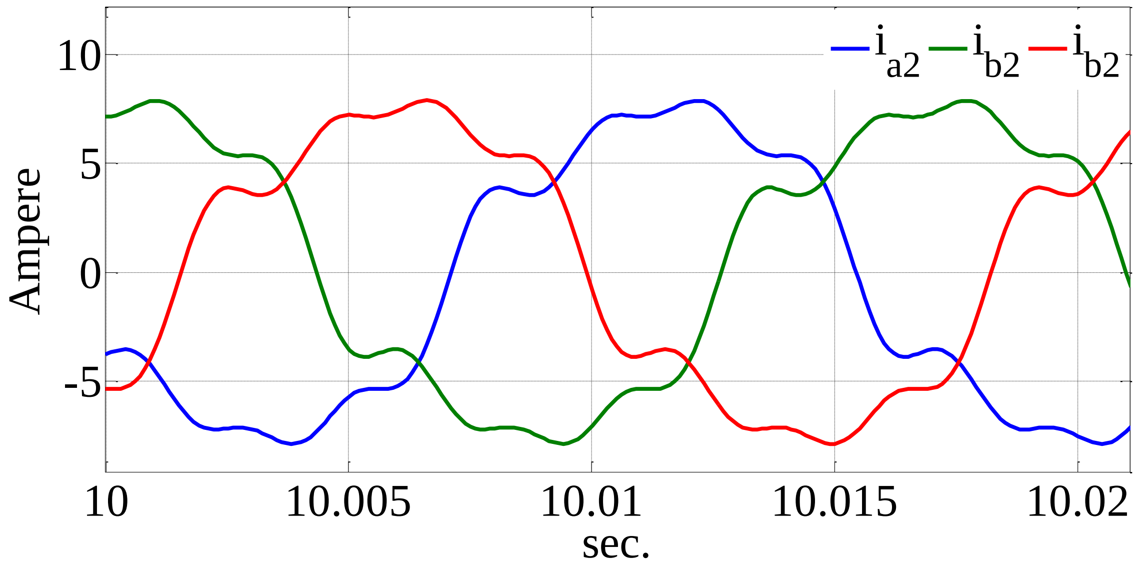


(a)

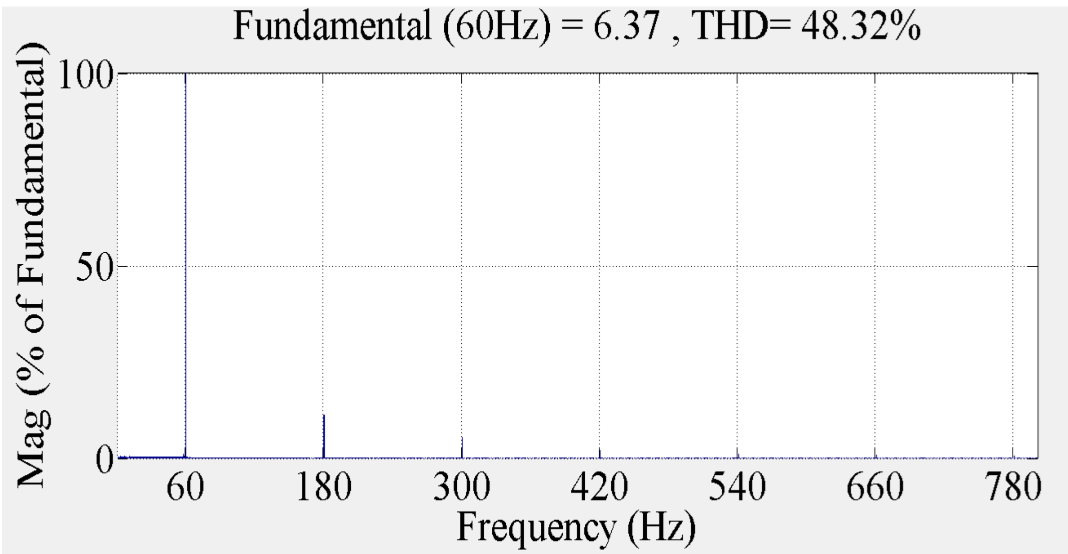


(b)

Figure 3.135: (a) The machine 1 stator currents at steady state, (b) The spectrum of the phase 'a' current.

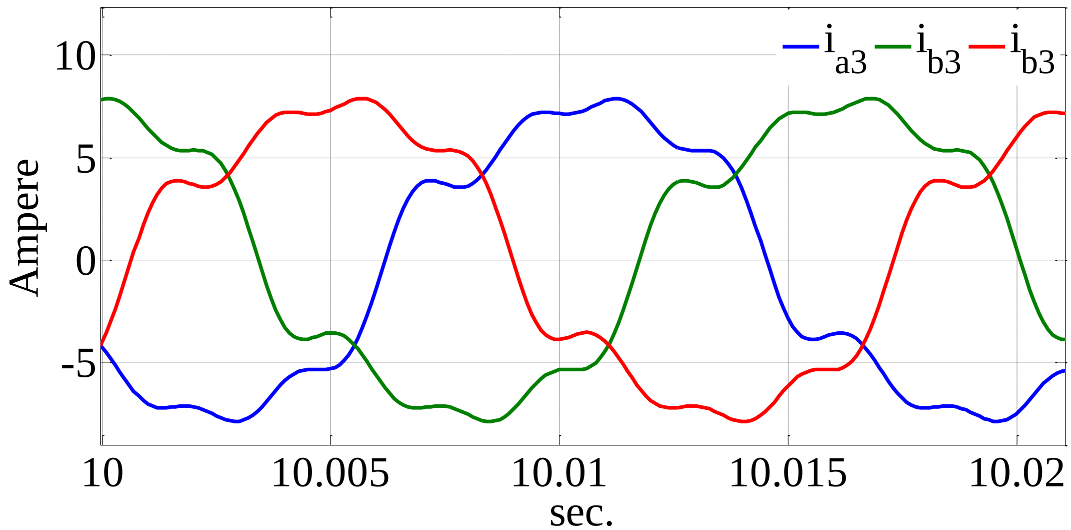


(a)

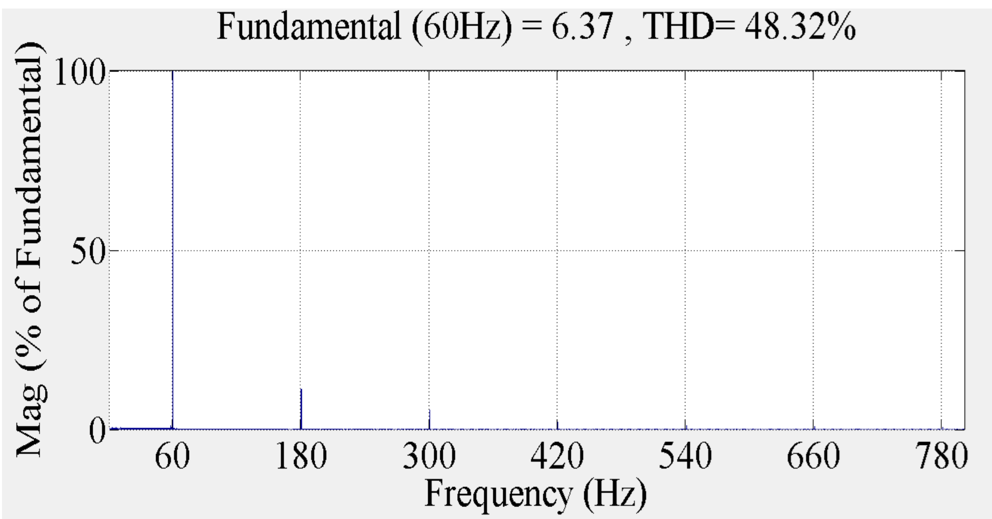


(b)

Figure 3.136: (a) The machine 2 stator currents at steady state, (b) The spectrum of the phase 'a' current.



(a)



(b)

Figure 3.137: (a) The machine 3 stator currents at steady state, (b) The spectrum of the phase 'a' current.

3.9 Conclusions

In this chapter coupled models of the nine phase IPM machine in different configurations are presented. The studied configurations include single star nine-phase, triple star symmetrical nine-phase and triple star asymmetrical nine-phase. These models use the basic geometry of the machine with limited simplifying assumptions. With limited simplifying assumptions, the models are accurate models to study the behavior of the machines when operating under different input voltages including high frequency signals. In this chapter after generating the turn functions of the stator windings, the winding functions of the machine are generated. Then using the turn functions, the winding functions and the air gap function, the inductances of each phase are generated. Then the generated inductances are transformed to rotor reference frame to obtain different inductances in rotor reference frame. The permanent magnet flux linkages are also generated and transformed to rotor reference frame and finally using the generated inductances the machines are modeled in the rotor reference frame. The generated models are simulated using MATLAB/Simulink and the Simulation results are presented.